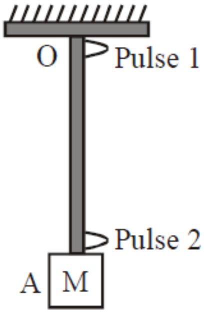
JEE ADVANCED-2017 PAPER-1

PHYSICS

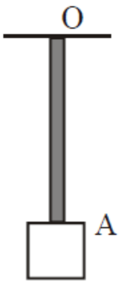
ONE OR MORE THAN ONE

1. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct ?



(A) The time $T_{AO} = T_{OA}$
(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
(C) The wavelength of Pulse 1 becomes longer when it reaches point A
(D) The velocity of any pulse along the rope is independent of its frequency and wavelength.

Ann. (AD)



(A) Speed of wave is property of medium so time taken to cross the string will be equal
(B) Speeds are same but velocity is vector, has opposite directions
(C) Wavelength $\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$ and $T_O > T_A$
(D) Velocity of any pulse is $v = \sqrt{\frac{T}{\mu}}$ and it is property of medium

2. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of $\sigma T_0^4 = 460\,wm^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct ?
(A) The amount of energy radiated by the body in 1 second is close to 60 Joules
(B) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time
(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
(D) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

Ans. (ABC) or (C)

Assumption : e = 1 (Black body radiation)

$P = \sigma A\left(T^4 - T_0^4\right)$

(A) $P_{rad} = \sigma A T^4 = \sigma .1.\left(T_0 + 10\right)^4$

$= \sigma \cdot T_0^4\left(1 + \frac{10}{T_0}\right)^4 \left(T_0 = 300 \text{ gives}\right)$

$= \sigma \cdot \left(300\right)^4 \cdot \left(1 + \frac{40}{300}\right) \approx 460 \times \frac{17}{15} \approx 520J$

$P_{net} = 520 - 460 \approx 60$ W

$\Rightarrow$ Energy radiated in 1 second = 60 Joule

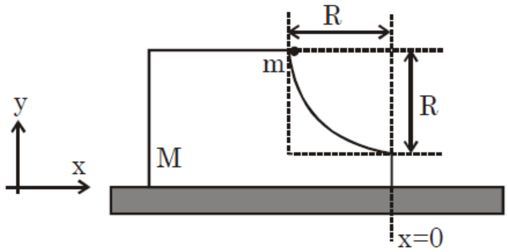
(B) $P = \sigma A\ (T^4 - T_0^4)$

$dP = \sigma A(0 - 4T_0^3 \cdot dT)$ & $dT = -\Delta T$

$\Rightarrow dP = 4\sigma A T_0^3\, \Delta T$

(C) Surface area decrease $\Rightarrow$ Energy radiation decreases

3. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct ?



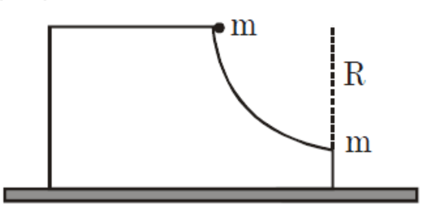
(A) The x component of displacement of the centre of mass of the block M is : $-\frac{mR}{M+m}$

(B) The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$

(C) The velocity of the point mass m is : $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

(D) The velocity of the block M is : $V = -\frac{m}{M}\sqrt{2gR}$

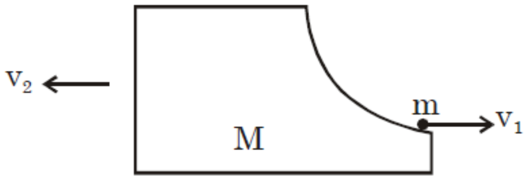
Ans. (AC)



$M_s \Delta \bar{x}_{cm} = m_1 \Delta \bar{x} + m_2 \Delta \bar{x}_2$

$0 = m\left(+R + \bar{x}\right) + m\bar{x}$

$\bar{x} = \frac{-mR}{M+m}$ (A) ans



$$0 = m\bar{v}_1 + M\bar{v}_2$$

$$\bar{v}_2 = -\frac{m\bar{v}_1}{M}$$

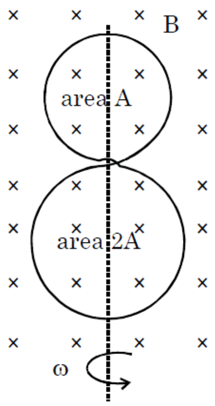
$$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{mv_1}{M}\right)^2$$

$$mgR = \frac{1}{2}mv_1^2\left(1+\frac{m}{M}\right)$$

$$\sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}} = v_1$$

4. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



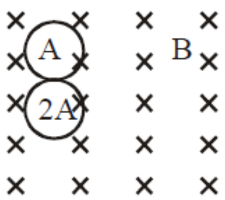
(A) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(B) The net emf induced due to both the loops is proportional to cos ωt

(C) The emf induced in the loop is proportional to the sum of the areas of the two loops

(D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

Ans. (AD)



$$\phi = |B||A|\cos\theta$$

$$= BA\cos(\omega t)$$

$$\varepsilon = -\frac{d\phi}{dt} = BA\omega\sin(\omega t)$$

so, ε & $\frac{d\phi}{dt} \propto \sin(\omega t)$

so, maximum when, $\omega t = \theta = \frac{\pi}{2}$.

Net emf will be difference of emfs in both loops because their polarities are opposite.

$\varepsilon_{Net} = \varepsilon_{2A} - \varepsilon_A - B(2A)\omega \sin \omega t - B(A)\omega \sin(\omega t)$

= B(2A – A)ωsinωt = BAωsinωt

5. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation δ_m = A. Which of the following options is/are correct ?

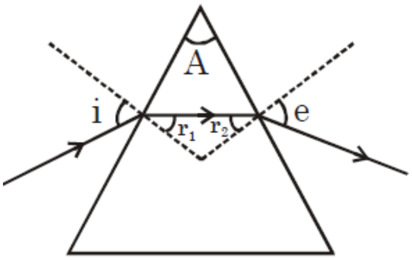
(A) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(B) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

(D) For the angle of incidence i_1 = A, the ray inside the prism is parallel to the base of the prism

Ans. (ACD)



i = e (for minimum deviation)

$r_1 + r_2 = A$, $r_1 = r_2$

(A) $\delta_m = 2i - A = A$ (given)

$\Rightarrow$ I = A

$\Rightarrow r_1 = \frac{A}{2} = \frac{i}{2}$

(B) $\mu = \frac{\sin(A)}{\sin(A/2)} = 2\cos\frac{A}{2} \Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) $\mu \sin(r_2) = 1$

$\sin(r_2) = \frac{1}{\mu}$

$r_1 + r_2 = A$

$r_1 = A - r_2$

$= A - \sin^{-1}\left[\frac{1}{\mu}\right]$

sin (i) = μsin (r_1)

$i = \sin^{-1}[\mu \sin[A - \sin^{-1}\left[\frac{1}{\mu}\right]]$

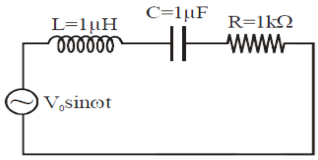
$i_g = \sin^{-1}\left[\sqrt{\mu^2 - 1}\sin A - \cos A\right] = \sin^{-1}\left[\mu \sin(A - \theta)\right]$

$\left(\text{Here } \mu = 2\cos\frac{A}{2}\right)$

(D) Condition of min. deviation i = e & $r_1 = r_2 = \frac{A}{2}$

Rays will be parallel to base.

6. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct ?



(A) The frequency at which the current will be in phase with the voltage is independent of R.
(B) At ω ~ 0 the current flowing through the circuit becomes nearly zero
(C) At $\omega >> 10^6$ rad.s^{-1}, the circuit behaves like a capacitor.
(D) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}.

Ans. (AB)

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega_0 = 10^6$ rad/s

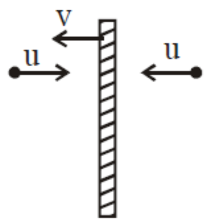
$$i_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

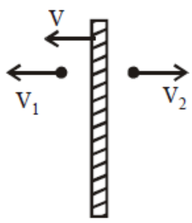
$\omega \to 0, i_0 \to 0$

For $\omega >> \omega_0$, circuit behaves as inductor.

7. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ?
(A) The resistive force experienced by the plate is proportional to v
(B) The pressure difference between the leading and trailing faces of the plate is proportional to uv.
(C) The plate will continue to move with constant non-zero acceleration, at all times
(D) At a later time the external force F balances the resistive force.

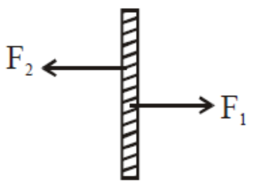
Ans. (ABD)





Just before the collision
$v_1 = u + 2v$
$\Delta v_1 = (2u + 2v)$
$F_1 = \frac{dp_1}{dt} = \rho A(u + v)(2u + 2v)$
$= 2\rho A(u + v)^2$

Just after the collision
$v_2 = (u - 2v)$
$\Delta v_2 = (2u - 2v)$
$F_2 = \frac{dp_2}{dt} = \rho A(u - v)(2u - 2v)$
$= 2\rho A(u - v)^2$



$\Delta F = F_1 - F_2$ (ΔF net force due to the air molecules on the plate)
$= 2\rho A\,(4uv) = 8\rho Auv$

$$P=\frac{\Delta F}{A}=8\rho(uv)$$

F_{net} = (F – ΔF) = ma (m is mass of the plate)

F – (8ρAu)v = ma

SINGLE DIGIT INTEGER

8. A drop of liquid of radius R = 10^{-2} m having surface tension $S=\frac{0.1}{4\pi}Nm^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy ΔU = 10^{-3} J. If K = 10^{α} then the value of α is

Ans. (6)

By mass conservation, $\rho.\frac{4}{3}\pi R^3=\rho.K.\frac{4}{3}\pi r^3$

$\Rightarrow\ R=K^{1/3}r$

$\therefore\ \Delta U=T\ \Delta A=T\left(K.4\pi r^2-4\pi R^2\right)$

$=T\left(K.4\pi R^2K^{-2/3}-4\pi R^2\right)$

$\Delta U = 4\pi R^2T[K^{1/3} - 1]$

Putting the value's $\Rightarrow\ 10^{-3}=\frac{10^{-1}}{4\pi}\times 4\pi\times 10^{-4}\left[K^{1/3}-1\right]$

$100 = K^{1/3} - 1$

$\Rightarrow\ K^{1/3}\cong 100=10^2$

Given that $K=10^{\alpha}\ \Rightarrow \therefore 10^{\alpha/3}=10^2$

$\Rightarrow\ \frac{\alpha}{3}=2$

$\Rightarrow\ \alpha=6$

9. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4 × 10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for |x| << 1 and ln 2 ≈ 0.7).

Ans. (5)

$t_{1/2}$ = 8 days

$A = A_0e^{-\lambda t}$

$e^{\lambda t}(115)=A_0$

$A_0 = 115(1 + \lambda t) = 115\left(1+\frac{\ell n2}{t_{1/2}}\times 11.5\right)$

A_0 = 115 × 1.042 = 119.82

$A_0 \cong 120$ Bq

120 Bq is the activity of 2.5 ml

$\therefore$ 2.4 × 10^5 Bq is the activity of $\frac{2.5\times 10^{-3}}{120}\times 2.4\times 10^5$

$\therefore$ Total volume of blood = 5 litres

10. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f}=6.25$, then the smallest possible n_f is.

Ans. (5)

$$U = -\frac{Z^2}{n^2}E_0$$

Z = 1

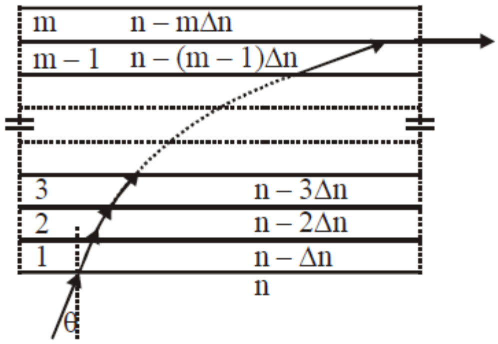
$$U = -\frac{E_0}{n^2}$$

$$\frac{U_i}{U_f} = \frac{n_f^2}{n_i^2} = 6.25$$

Taking $n_i = 2$

$n_f = 5$

11. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as nm = n – mΔn, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m ?



Ans. (8)

Applying snell's law between entry & exit surfaces,

$$n\sin\theta = \mu\sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow\ 1.6\sin 30° = \mu\sin\left(\frac{\pi}{2}\right)$$

$\therefore\ \mu = 0.8$

$\therefore\ 0.8 = n - m\Delta n$

$= 1.6 - m \times 0.1$

$\therefore\ m = 8$

12. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz ? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received).

Ans. (6)

Frequency of sound as received by large car approaching the source.

$$f_1 = \frac{C + V_0}{C} f_0 = \left(\frac{330 + 2}{330}\right) 492\,\text{Hz}$$

This car now acts as source for reflected sound wave

$\therefore\ F_{reflected} = f_1$

frequency of sound received by source,

$$f_2 = \left(\frac{C}{C - V_0}\right) f_{reflected} = \left(\frac{330}{330 - 2}\right) \times f_1 = \frac{330}{328} \times \frac{332}{330} \times 492\,\text{Hz}$$

$$\therefore\ \text{Beat frequency } = |f_0 - f_2| = \left(\frac{332}{328} - 1\right) \times 492\,\text{Hz}$$

= 6 Hz

MATCHING TYPE (13 – 15)

An ideal gas is undergoing a cyclic thermodynamics process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamics processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column – I	Column – II	Column – III
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

13. In which case will the particle move in a straight line with constant velocity ?

(A) (II) (iii) (S) (B) (IV) (i) (S) (C) (III) (ii) (R) (D) (III) (iii) (P)

Ans. (A)

$\vec{F}_{net} = \vec{F}_e + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B}$

For particle to move in straight line with constant velocity, $\vec{F}_{net} = 0$

$\therefore \ q\vec{E} + q\vec{v} \times \vec{B} = 0$

14. In which case will the particle describe a helical path with axis along the positive z-direction ?

(A) (II) (ii) (R) (B) (IV) (ii) (R) (C) (IV) (i) (S) (D) (III) (iii) (P)

Ans. (C)

For path to be helix with axis along +ve z-direction, particle should experience a centripetal acceleration in x-y plane.

For the given set of options only option (C) satisfy the condition. Path is helical with increasing pitch.

15. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$) ?

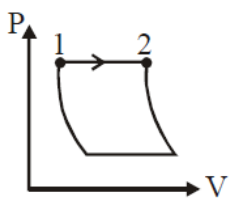
(A) (IV) (ii) (S) (B) (III) (ii) (P) (C) (II) (iii) (Q) (D) (III) (ii) (R)

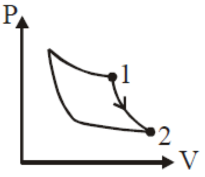
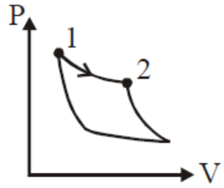
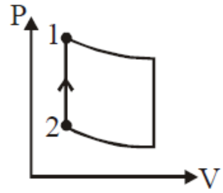
Ans. (D)

For particle to move in -ve y-direction, either its velocity must be in –ve y-direction (if initial velocity ≠ 0) & force should be parallel to velocity or it must experience a net force in –ve y-direction only (if initial velocity = 0)

MATCHING TYPE (16 – 18)

An ideal gas is undergoing a cyclic thermodynamics process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamics processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column – I	Column – II	Column – III
(I) $W_{1\to2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$	(i) Isothermal	 (P)

(II) $W_{1\to 2} = -PV_2 + PV_1$	(ii) Isochoric	 (Q)
(III) $W_{1\to 2} = 0$	(iii) Isobaric	 (R)
(IV) $W_{1\to 2} = -nRT\ln\frac{V_2}{V_1}$	(iv) Adiabatic	 (S)

16. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?
(A) (II) (iv) (R) (B) (II) (iii) (P) (C) (II) (iii) (S) (D) (III) (iii) (P)

Ans. (B)
Work (Column-I), process (Column-II) & corresponding graph (Column-III) are in this sequence.

$I \longrightarrow IV \longrightarrow Q$

$II \longrightarrow III \longrightarrow P$

$III \longrightarrow II \longrightarrow S$

$IV \longrightarrow I \longrightarrow R$

Only "B" option follow the sequence.

17. Which one of the following options is the correct combination ?
(A) (III) (ii) (S) (B) (II) (iv) (R) (C) (II) (iv) (P) (D) (IV) (ii) (S)

Ans. (A)
Only option "A" follow the sequence.

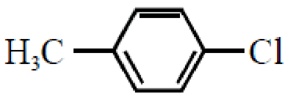
18. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ?
(A) (III) (iv) (R) (B) (I) (ii) (Q) (C) (IV) (ii) (R) (D) (I) (iv) (Q)

Ans. (D)
It is for an adiabatic process. Only option (D).

CHEMISTRY

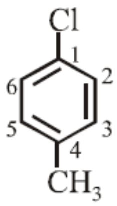
ONE OR MORE THAN ONE

19. The IUPAC name(s) of the following compound is(are)



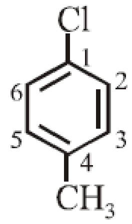
(A) 4-methylchlorobenzene (B) 4-chlorotoluene
(C) 1-chloro-4-methylbenzene (D) 1-methyl-4-chlorobenzene

Ans. (BC)



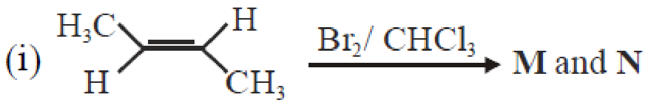
1-Chloro-4-methylbenzene

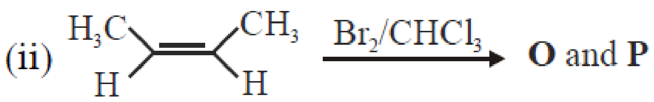
IUPAC Name- "Toluene" is accepted by IUPAC as a name of parent carbon chain.



So it can also be named as 4-chlorotoluene.

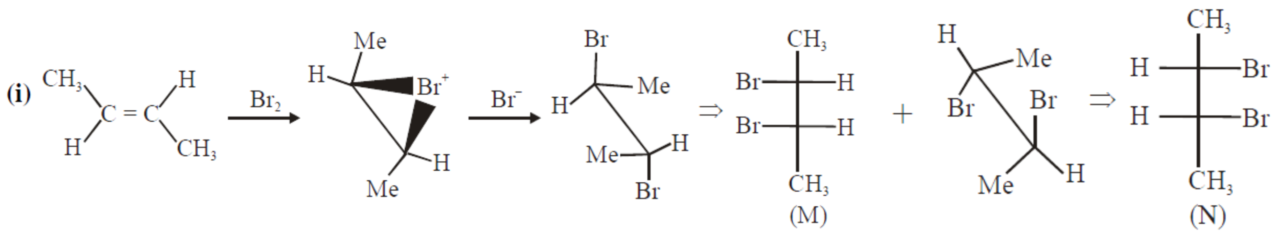
20. The correct statement(s) for the following addition reactions is(are)



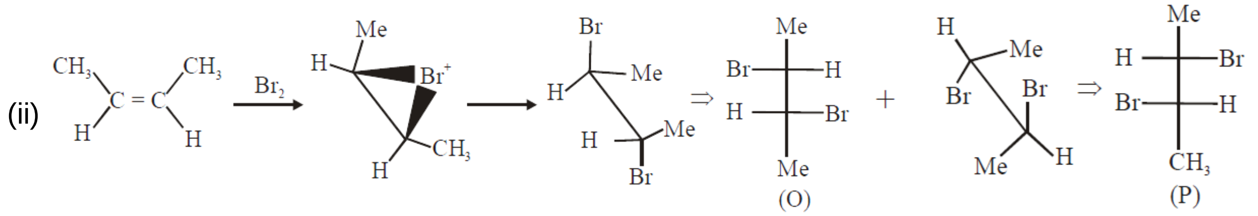


(A) (**M** and **O**) and (**N** and **P**) are two pairs of diastereomers
(B) Bromination proceeds through trans-addition in both the reactions
(C) **O** and **P** are identical molecules
(D) (**M** and **O**) and (**N** and **P**) are two pairs of enantiomers

Ans. (AB)



(M) and (N) are identical meso compounds



(O) and (P) are enantiomers

Explanation of 4 options :
(A) (M) and (O) are distereomers of each other.
(N) and (P) are distereomers of each other.
(B) Addition of Br2 on alkene follows non-classical carbocation mechanism. It is anti or trans addition.
(C) (O) and (P) are enantiomers
(D) (M) and (N) are identical and (O) and (P) are enantiomers.
(M and O) are distereomers and (N and P) are distereomers.

21. Addition of excess aqueous ammonia to a pink coloured aqueous solution of $MCl_2 \cdot 6H_2O$ (X) and NH_4Cl gives an octahedral complex Y in the presence of air. In aqueous solution, complex Y behaves as 1 : 3 electrolyte. The reaction of X with excess HCl at room temperature results in the formation of a blue coloured complex Z. The calculated spin only magnetic moment of X and Z is 3.87 B.M., whereas it is zero for complex Y.
Among the following options, which statements is(are) correct ?
(A) The hybridization of the central metal ion in Y is d^2sp^3
(B) Z is tetrahedral complex

(C) Addition of silver nitrate to Y gives only two equivalents of silver chloride
(D) When X and Z are in equilibrium at 0°C, the colour of the solution is pink

Ans. (ABD)

$$\overset{II}{[Co(H_2O)_6]Cl_2} \xrightarrow[\text{Air / } O_2]{\text{Excess } NH_4OH / NH_4Cl} \overset{III}{[Co(NH_3)_6]Cl_3}$$

Pink **(X)** **(Y)**

$$[Co(H_2O)_6]^{2+} + 4Cl^- \longrightarrow [CoCl_4]^{2-}$$

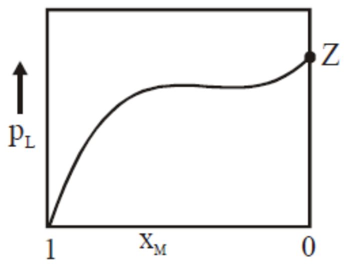
(X) (excess) **(Z)** blue colour

(A) Hybridisation of (Y) is d^2sp^3 as NH_3 is strong field ligand
(B) $[CoCl_4]^{2-}$ have sp^3 hybridisation as Cl^- is weak field ligand
(C) $\overset{III}{[Co(NH_3)_6]Cl_3} + 3AgNO_3(aq.) \rightarrow 3AgCl$
(Y)
(D) $\underset{(Blue)}{[CoCl_4]^{2-}} + 6H_2O \rightleftharpoons \underset{(Pink)}{[Co(H_2O)_6]^{2+}} + 4Cl^- \quad \Delta H = (-)ve \ \ (exothermic)$

When ice is added to the solution the equilibrium shifts right hence pink colour will remain predominant

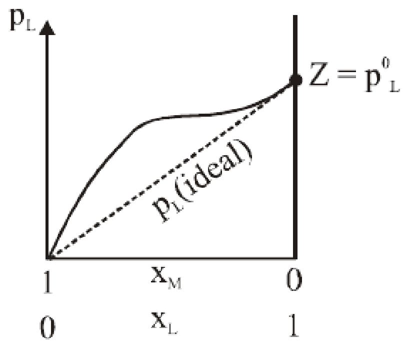
So, correct answer is (A,B & D)

22. For a solution formed by mixing liquids L and M, the vapour pressure of L plotted against the mole fraction of M in solution is shown in the following figure, Here x_L and x_M represent mole fractions of L and M, respectively, in the solution. the correct statement(s) applicable to this system is(are)



(A) Attractive intramolecular interactions between L–L in pure liquid L and M–M in pure liquid M are stronger than those between L–M when mixed in solution
(B) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed when $x_L \rightarrow 0$
(C) The point Z represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \rightarrow 1$
(D) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed from $x_L = 0$ to $x_L = 1$

Ans. (AC)



(A) This is case of positive deviation hence
$F_{L-L}, F_{M-M} > F_{L-M}$
(C) $P_L \geq P_L^0 X_L$
but when $X_L \rightarrow 1$, mixture has almost pure liquid L so, $P_L \rightarrow P_L^0$

23. An ideal gas is expanded from (p_1, V_1 , T_1) to (p_2 , V_2 , T_2) under different conditions. The correct statement(s) among the following is(are)

(A) The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1

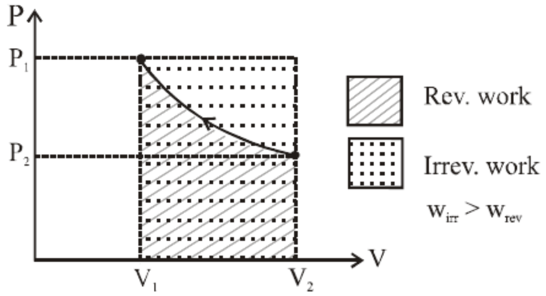
(B) The work done on the gas is less when it is expanded reversibly from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversibly from V_1 to V_2 under isothermal conditions.

(C) The change in internal energy of the gas (i) zero, if it is expanded reversibly with $T_1 = T_2$, and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$

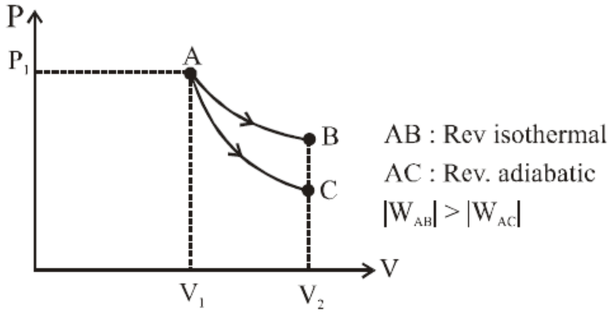
(D) If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic

Ans. (ABD)

(A)



(B)



(C) (i) $\Delta U = nC_v \Delta T = 0$ (isothermal hence $\Delta T = 0$)

(ii) $\Delta U = q + w = -ve$ ($q = 0$, $w < 0$)

$\Delta U = nC_v \Delta T \Rightarrow \Delta T < 0$

(D) $q = 0$ (adiabatic), $w = 0$ (free expansion)

$\Delta U = 0 \Rightarrow \Delta T = 0$ (isothermal)

24. The correct statements(s) about the oxoacids, $HClO_4$ and HClO, is (are)

(A) $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

(B) $HClO_4$ is formed in the reaction between Cl_2 and H_2O

(C) The central atom in Both $HClO_4$ and HClO is sp^3 hybridized

(D) The conjugate base of $HClO_4$ is weaker base than H_2O

Ans. (ACD)

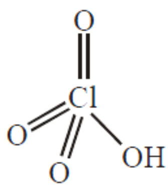
(A) $HClO_4 \rightleftharpoons H^+ + ClO_4^-$ ClO_4^- is resonance stablized anion

$HClO \rightleftharpoons H^+ + ClO^-$

$\therefore$ $HClO_4$ is more acidic then HClO.

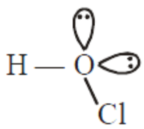
(B) $Cl_2 + H_2O \longrightarrow HClO + HCl$

(C) $HClO_4$



Central atom is sp^3 hybridised

HClO



Central atom is sp^3 hybridised

(D) $HClO_4 + H_2O \rightarrow H_3O^{\oplus} + ClO_4^-$ since H_2O is accepting H^+ from $HClO_4$ so H_2O is stronger base compare to ClO_4^-

25. The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to
(A) the physical state of X_2 at room temperature changes from gas to solid down the group
(B) decrease in HOMO-LUMO gap down the group
(C) decrease in π^*-σ^* down the group
(D) decrease in ionization energy down the group

Ans. (BC)
Halogens are coloured due to HOMO-LUMO transition of electrons.

$$\underset{\text{(configuration)}}{\text{M.O.}} \rightarrow F_2 = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \overset{*}{\pi} 2p_x^2 = \overset{*}{\pi} 2p_y^2, \overset{*}{\sigma} 2p_z$$

filled (HOMO) — Vacant (LUMO)

On moving down the group HOMO-LUMO energy gap decreases so transition of electrons become $\overset{*}{\pi}2p$ to $\overset{*}{\sigma}2p$ therefore colour intensify.

SINGLE DIGIT INTEGER

26. Among H_2, He_2^+, Li_2, Be_2, B_2, C_2, N_2, O_2^-, and F_2, the number of diamagnetic species is
(Atomic number) : H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8 , f = 9)

Ans. (5 or 6)

$H_2 \Rightarrow \sigma 1s^2$ (Diamagnetic)

$He_2^{\oplus} \Rightarrow \sigma 1s^2\ \sigma^* 1s^1$ (Paramagnetic)

$Li_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2$ (Diamagnetic)

$Be_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2$ (Diamagnetic)

$B_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^1 = \pi 2p_y^1$ (Paramagnetic)

$C_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2$ (Diamagnetic)

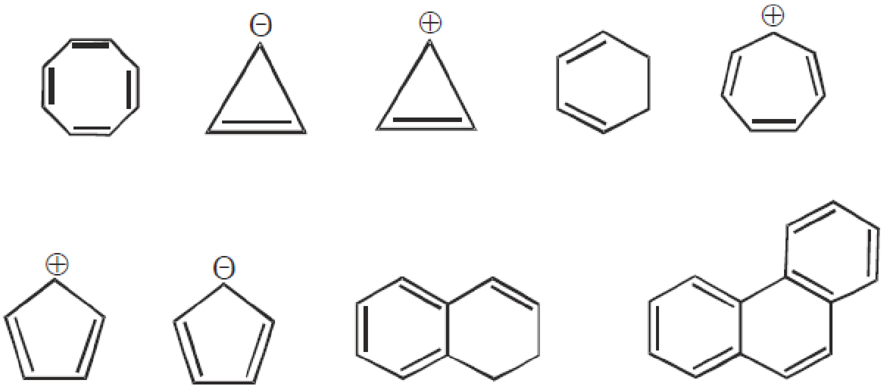
$N_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \sigma 2p_z^2$ (Diamagnetic)

$O_2^{\ominus} \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2 = \pi 2p_y^2\ \pi^* 2p_x^2 = \pi^* 2p_y^1$ (Paramagnetic)

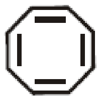
$F_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2 = \pi 2p_y^2\ \pi^* 2p_x^2 = \pi^* 2p_y^2$ (Diamagnetic)

If existence of Be_2 is considered in atomic form or very weak bonded higher energetic species having zero bond order then it is diamagnetic, then answer will be 6. But if existence of molecular form of Be_2 is not considered then magnetic property can't be predicted then answer will be 5.

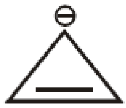
27. Among the following, the number of aromatic compound (s) is



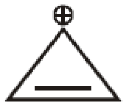
Ans. (5)



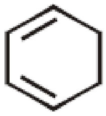
Cyclooctatetraene ; non aromatic
Due to nonplanarity of ring the π-electrons are not delocalised.



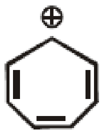
Cyclopropenyl anion ; Anti aromatic
4π-electrons delocalised.



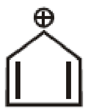
Cyclopropenyl cation ; Aromatic
2π-electrons delocalised.



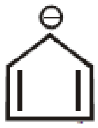
Cyclohexadiene : Non-aromatic



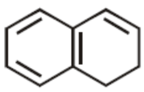
Tropylium ion : Aromatic
6π-electrons delocalised.



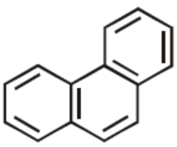
Cyclo pentadienyl cation ; Anti-aromatic
4π-electrons delocalised.



Cyclo pentadienyl anion ; Aromatic
6π-electrons delocalised.



aromatic



aromatic
$14\ \pi e^-$s
(delocalised)

28. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be 5×10^{-7}S. The pH of the solution is 4. The value of limiting molar conductivity $\left(\Lambda_m^0\right)$ of this weak monobasic acid in aqueous solution is Z × 10^2S $cm^{-1}mol^{-1}$. The value of Z is.

Ans. (6)

For weak acid $[H^+] = c\alpha = c\dfrac{\Lambda_m^C}{\Lambda_m^\infty} = c \times \dfrac{\kappa \times \dfrac{1000}{c}}{\Lambda_m^\infty} = \dfrac{\kappa \times 1000}{\Lambda_m^\infty} = \dfrac{G \times \left(\dfrac{\ell}{a}\right) \times 1000}{\Lambda_m^\infty}$

$$10^{-4}=\frac{5\times10^{-7}\times\left(\frac{120}{1}\right)\times1000}{Z\times10^{2}}\Rightarrow Z=6$$

29. The sum of the number of lone pairs of electrons on each central atom in the following species is.
$[TeBr_6]^{2-}$, $[BrF_2]^{+}$, SNF_3 and $[XeF_3]^{-}$
[Atomic number : N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54]

Ans. (6)

		Number of σ-bonds formed by central atom	Number of lone pairs on central atom
(i)	In $[TeBr_6]^{2-}$	6	1
(ii)	In $[BrF_2]^{+}$	2	2
(iii)	In SNF_3	4	0
(iv)	In $[XeF_3]^{-}$	3	3

$\Rightarrow$ Total number of lone pairs of electrons = 1 + 2 + 0 + 3 = 6

30. A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 400 pm. If the density of the substance in the crystal is 8g cm^{-3}, then the number of atoms present in 256g of the crystal is $N \times 10^{24}$. The value of N is

Ans. (2)

Formula of density $=\frac{Z\times M}{N_A\times a^3}$

For FCC unit cell Z = 4

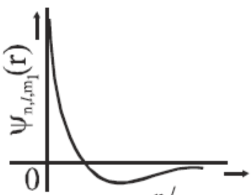
Edge length $a = 4 \times 10^{-8}$ cm

$$M=\frac{d\times N_A\times a^3}{Z}=\frac{8\times6\times10^{23}\times64\times10^{-24}}{4}\text{gm/mol}$$

No. of atoms $=\frac{\text{wt(gm)}}{\text{molar mass}}\times N_A=\frac{256\times10\times6\times10^{23}}{8\times6\times16}=2\times10^{24}$ (Value of N = 2)

MATCHING TYPE (31 – 33)

The wave function ψ_{n,ℓ,m_1} is a mathematical function whose value depends upon spherical polar coordinates (r, θ, ϕ) of the electron and characterized by the quantum numbers n, l and m_1. Here r is distance from nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the Table, Z is atomic number a_0 is Bohr radius.

Column – I	Column – II	Column – III
(I) 1s orbital	(i) $\psi_{n,\ell,m_1}\propto\left(\frac{Z}{a_0}\right)^{3/2}e^{-\left(\frac{Zr}{a_e}\right)}$	 (P)
(II) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto\frac{1}{a_0^3}$
(III) $2p_z$ orbital	(iii) $\psi_{n,\ell,m_1}\propto\left(\frac{Z}{a_0}\right)^{5/2}re^{-\left(\frac{Zr}{2a_0}\right)}\cos\theta$	(R) Probability density is maximum at nucleus
(IV) $3d_z^2$ orbital	(iv) xy - plane is a nodal plane	(S) Energy needed to excite electron from n = 2 state to n = 4 state is $\frac{27}{32}$ times the energy needed to excite electron from n = 2 state to n = 6 state

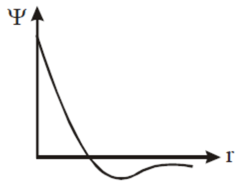
31. For the given orbital in column 1, the only CORRECT combination for any hydrogen - like species is :

(A) (IV) (iv) (R) (B) (II) (ii) (P) (C) (III) (iii) (P) (D) (I) (ii) (S)

Ans. (B)

(A) (IV) (iv) (R) $\Rightarrow$ incorrect, because, d_{z^2} has no nodal plane.

(B) (II) (ii) (P) $\Rightarrow$ correct, because 2s orbtial has 1 radial node.



(C) (III) (iii) (P) $\Rightarrow$ incorrect, because probability density for 2p at nucleus is zero.

(D) (I) (ii) (S) $\Rightarrow$ incorrect, because 1s orbital has no radial node.

32. For He^+ ion, the only INCORRECT combination is

(A) (II) (ii) (Q) (B) (I) (i) (S) (C) (I) (i) (R) (D) (I) (iii) (R)

Ans. (D)

The option (D) is incorrect because in the wave function of 1s orbital , no angular function should be present.

33. For hydrogen atom, the only CORRECT combination is

(A) (I) (iv) (R) (B) (I) (i) (P) (C) (II) (i) (Q) (D) (I) (i) (S)

Ans. (D)

We have to select only correct combination hence, the option (D) is correct.

For 1s orbital : $\psi_{n,\ell,m} \alpha \left(\frac{Z}{a_0}\right)^{3/2} e^{-\frac{zr}{a_0}}$

Energy needed to excite : from n = 2 to n = 4

$$\Delta E_{2-4} = 13.6\ Z^2 \times \frac{3}{16} eV$$

Energy needed to excite from : n = 2 to n = 6

$$\Delta E_{2-6} = 13.6\ Z^2 \times \frac{8}{36}$$

$$\Delta E_{2-4} = \frac{27}{32} E_{2-6} \quad \text{(hence, true)}$$

MATCHING TYPE (34 – 36)

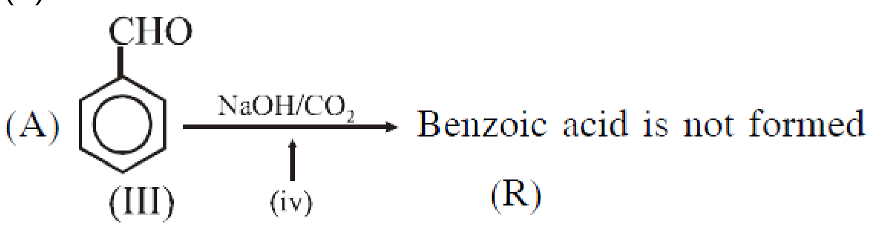
Columns 1, 2 and 3 contains starting materials, reaction conditions, , and type of reactions, respectively .

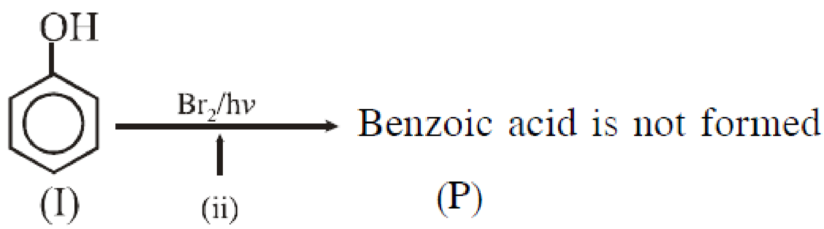
Column – I	Column – II	Column – III
(I) Toluene	(i) $NaOH/Br_2$	(P) Condensation
(II) Acetophenone	(ii) Br_2 / hv	(Q) Carboxylation
(III) Banzaldehyde	(iii) $(CH_3CO)_2O/CH_3COOK$	(R) Substitution
(IV) Phenol	(iv) $NaOH/CO_2$	(S) Haloform

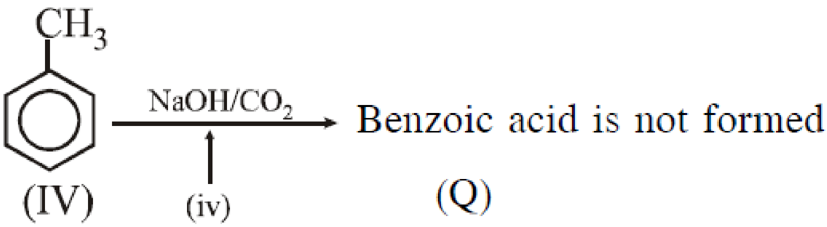
34. For the synthesis of benzoic acid, the only CORRECT combination is

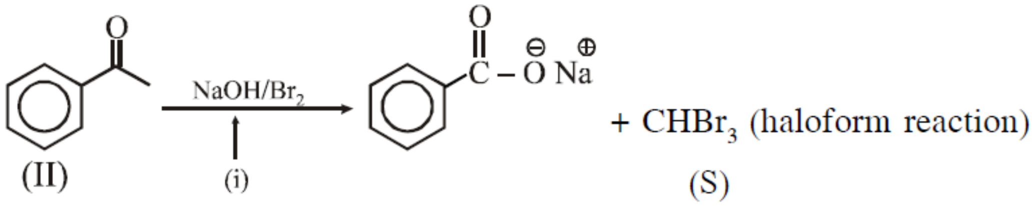
(A) (III) (iv) (R) (B) (IV) (ii) (P) (C) (I) (iv) (Q) (D) (II) (i) (S)

Ans. (D)



(B) 

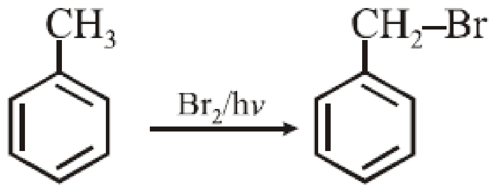
(C) 

(D) 

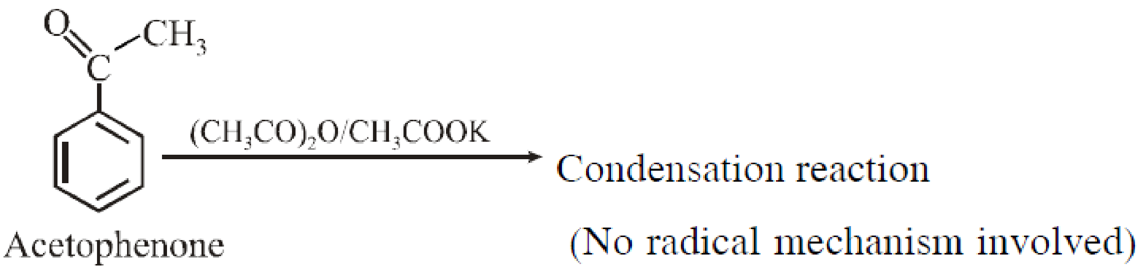
35. The only CORRECT combination in which the reaction proceeds through radical mechanism is

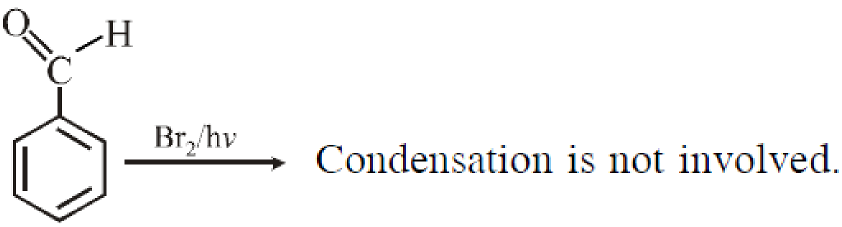
(A) (I) (ii) (R) (B) (II) (iii) (R) (C) (III) (ii) (P) (D) (IV) (i) (Q)

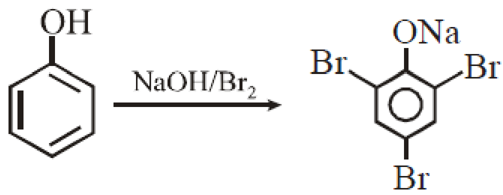
Ans. (A)



mechanism involved is free radical substitution

(B) 

(C) 

(D) 

(No free radical mechanism)

36. The only CORRECT combination that gives two different carboxylic acids is

(A) (IV) (iii) (Q) (B) (III) (iii) (P) (C) (II) (iv) (R) (D) (I) (i) (S)

Ans. (B)

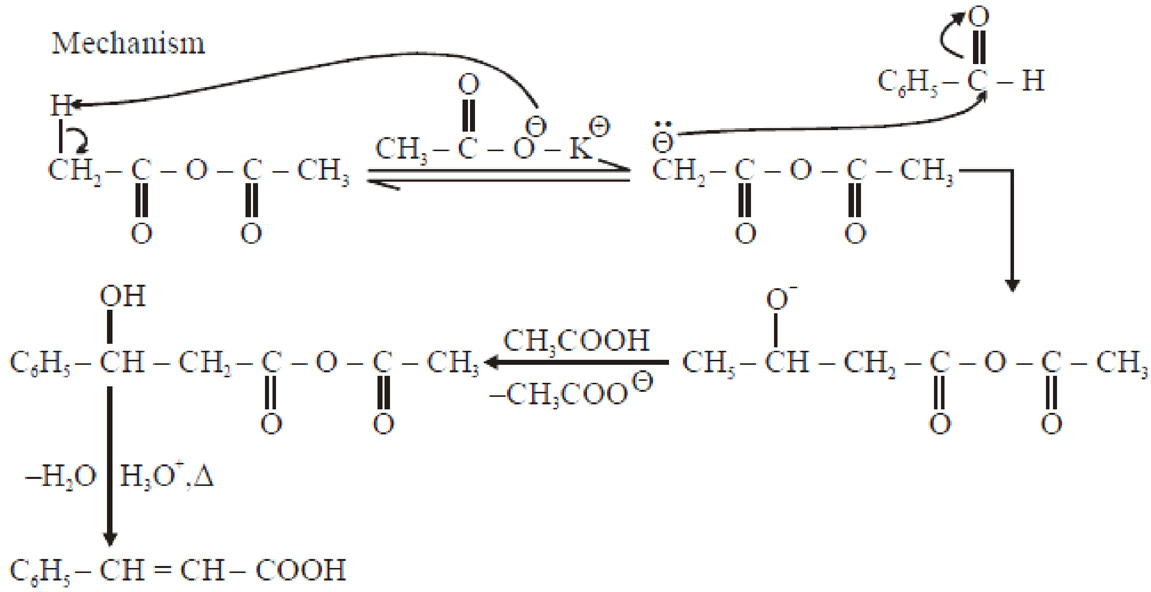
$$C_6H_5CHO + CH_3-\underset{\underset{O}{\|}}{C}-O-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{CH_3COOK} C_6H_5CH=CH-COOH$$

Cinnamic acid

Cis and Trans

It is perkin condensation reaction

Mechanism



Cis / Trans

MATHS

ONE OR MORE THAN ONE

37. Which of the following is(are) NOT the square of a 3 × 3 matrix with real entries ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Ans. (AD)

38. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?

(A) p = 5, h = 4, k = –3
(B) p = –1, h = 1, k = –3
(C) p = –2, h = 2, k = –4
(D) p = 2, h = 3, k = –4

Ans. (D)

Equation of chord with mid point (h, k) :

$k.y - 16\left(\frac{x+h}{2}\right) = k^2 - 16h$

$\Rightarrow 8x - ky + k^2 - 8h = 0$

Comparing with $2x + y - p = 0$, we get

$k = -4;\ 2h - p = 4$

only (D) satisfies above relation.

39. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible values(s) of x ?

(A) $-1-\sqrt{1-y^2}$ (B) $1+\sqrt{1+y^2}$ (C) $1-\sqrt{1+y^2}$ (D) $-1+\sqrt{1-y^2}$

Ans. (AD)

$\text{Im}\left(\frac{az+b}{z+1}\right) = y$ and $z = x + iy$

$\therefore\ \text{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$

$\Rightarrow \text{Im}\left(\frac{(ax+b+iay)(x+1-iy)}{(x+1)^2+y^2}\right) = y$

$\Rightarrow -y(ax + b) + ay(x + 1) = y((x + 1)^2 + y^2)$

$\Rightarrow (a - b)y = y((x + 1)^2 + y^2)$

$\because\ y \neq 0$ and $a - b = 1$

$\Rightarrow (x + 1)^2 + y^2 = 1$

$\Rightarrow x = -1 \pm \sqrt{1-y^2}$

40. Let X and Y be two events such that $P(X) = \frac{1}{3}, P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

(A) $P(X'|Y) = \frac{1}{2}$ (B) $P(X \cap Y) = \frac{1}{5}$ (C) $P(X \cup Y) = \frac{2}{5}$ (D) $P(Y) = \frac{4}{15}$

Ans. (AD)

$P(x) = \frac{1}{3};\ \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2};\ \frac{P(Y \cap X)}{P(X)} = \frac{2}{5}$

from this information, we get

$P(X \cap Y) = \frac{2}{15};\ P(Y) = \frac{4}{15}$

$\therefore\ P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$

$$P\left(\bar{X}/Y\right)=\frac{P\left(\bar{X}\cap Y\right)}{P(Y)}=\frac{P(Y)-P(X\cap Y)}{P(Y)}$$

$$\Rightarrow P\left(\bar{X}/Y\right)=1-\frac{2/15}{4/15}=\frac{1}{2}$$

41. Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ?

(A) $x = -1$ (B) $x = 0$ (C) $x = 2$ (D) $x = 1$

Ans. (ACD)

$f(x) = x\cos(\pi x + [x]\pi)$

$\Rightarrow f(x) = (-1)^{[x]}\, x\cos\pi x$.

Discontinuous at all integers except zero.

42. If $2x - y + 1 = 0$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{16}=1$, then which of the following CANNOT be sides of a right angled triangle ?

(A) 2a, 4, 1 (B) 2a, 8, 1 (C) a, 4, 1 (D) a, 4, 2

Ans. (BCD)

The line $y = mx + c$ is tangent to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, if $c^2 = a^2m^2 - b^2$

$$\therefore\ (1)^2=4a^2-16\Rightarrow a^2=\frac{17}{4}$$

$$\Rightarrow a=\frac{\sqrt{17}}{2}$$

For option (A), sides are $\sqrt{17},4,1$ ($\Rightarrow$ Right angled triangle)

For option (B), sides are $\sqrt{17},8,1$ ($\Rightarrow$ Triangle is not possible)

For option (C), sides are $\frac{\sqrt{17}}{2},4,1$ ($\Rightarrow$ Triangle is not possible)

For option (D), sides are $\frac{\sqrt{17}}{2},4,2$ ($\Rightarrow$ Triangle exist but not right angled)

43. Let $f : R \to (0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

(A) $e^x-\int_0^x f(t)\sin t\,dt$ (B) $x^9 - f(x)$ (C) $f(x)+\int_0^{\pi/2} f(t)\sin t\,dt$ (D) $x-\int_0^{\pi/2-x} f(t)\cos t\,dt$

Ans. (BD)

For option (A),

Let $g(x)=e^x-\int\limits_0^x f(t)\sin t\,dt$

$\therefore\ g'(x)=e^x-\left(f(x).\sin x\right)>0\ \forall x\in(0,1)$

$\Rightarrow$ g(x) is strictly incrasing function.

Also, g(0) = 1

$\Rightarrow g(x) > 1\ \forall\ x \in (0,1)$

$\therefore$ option (A) is not possible.

For option (B), let

$k(x) = x^9 - f(x)$

Now, $k(0) = -f(0) < 0$ (As $f \in (0,1)$)

Also, $k(1) = 1 - f(1) > 0$ (As $f \in (0,1)$)

$\Rightarrow k(0).\ k(1) < 0$

So, option(B) is correct.

For option (C), let

$$T(x)=f(x)+\int\limits_0^{\pi/2} f(t).\sin t\,dt$$

$\Rightarrow T(x) > 0 \forall x \in (0,1)$ (As f $\in$ (0, 1))

so, option(C) is not possible.

For option (D),

Let $M(x) = x - \int_0^{\pi/2-x} f(t)\cos t\,dt$

$\therefore M(0) = 0 - \int_0^{\pi/2} f(t).\cos t\,dt < 0$

Also, $M(1) = 1 - \int_0^{\pi/2-1} f(t).\cos t\,dt > 0$

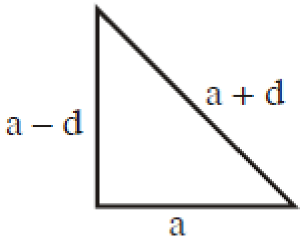
$\Rightarrow$ M(0). M(1) < 0

$\therefore$ option (D) is correct.

SINGLE DIGIT INTEGER

44. The sides of the right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

Ans. (6)



where d > 0, a > 0

$\Rightarrow$ length of smallest side = a – d

Now $(a + d)^2 = a^2 + (a - d)^2$

$\Rightarrow$ a(a – 4d) = 0

$\therefore$ a = 4d ...(1)

(As a = 0 is rejected)

Also, $\frac{1}{2}a.(a-d) = 24$

$\Rightarrow$ a(a – d) = 48 ...(2)

$\therefore$ From (1) and (2), we get a = 8, d = 2

Hence, length of smallest side

$\Rightarrow$ (a – d) = (8 – 2) = 6

45. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?

Ans. (2)

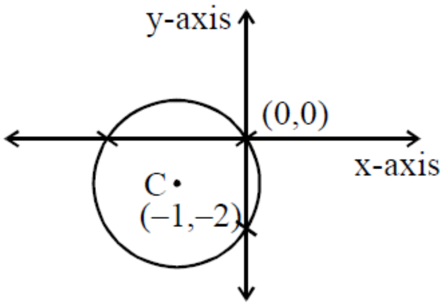
We shall consider 3 cases.

Case I : When p = 0

(i.e. circle passes through origin)

Now, equation of circle becomes

$x^2 + y^2 + 2x + 4y = 0$



Case II : When circle intersects x-axis at 2 distinct points and touches y-axis

Now $(g^2 - c) > 0$ & $f^2 - c = 0$

$\Rightarrow 1 - (-p) > 0$ & $4 - (-p) = 0 \Rightarrow p = -4$

$\Rightarrow p > -1$

$\therefore$ Not possible.

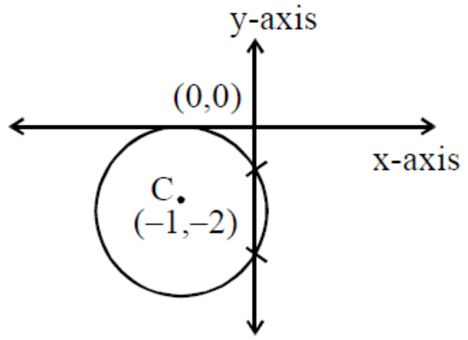
Case III : When circle intersects y-axis at 2 distinct points & touches x-axis.

Now, $g^2 - c = 0$ & $f^2 - c > 0$

$\Rightarrow 1 - (-p) = 0$ & $4 - (-p) > 0$

$\Rightarrow p = -1 \Rightarrow p > -4$

$\therefore p = -1$ is possible.



$\therefore$ Finally we conclude that p = 0, –1

$\Rightarrow$ Two possible values of p.

46. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Ans. (1)

$$\Delta = 0 \Rightarrow 1(1-\alpha^2) - \alpha(\alpha - \alpha^3) + \alpha^2(\alpha^2 - \alpha^2) = 0$$

$$(1-\alpha^2) - \alpha^2 + \alpha^4 = 0$$

$$(\alpha^2 - 1)^2 = 0 \Rightarrow \alpha = \pm 1$$

but at $\alpha = 1$ No solution so rejected

at $\alpha = -1$ all three equation become

x – y + z = 1 (coincident planes)

$\therefore 1 + \alpha + \alpha^2 = 1$

47. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

Ans. (5)

x = 10!

$$y = {}^{10}C_1 {}^{9}C_8 \frac{10!}{2!}$$

$$\frac{y}{9x} = \frac{5.9.10!}{9.10!} = 5$$

48. Let f : R → R be a differentiable function such that $f(0) = 0, f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\pi/2} \left[f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)\right] dt \text{ for } x \in \left(0, \frac{\pi}{2}\right], \text{ then } \lim_{x\to 0} g(x) =$$

Ans. (2)

$$g(x)=\int_x^{\pi/2}\left(f'(t)\operatorname{cosec} t - f(t)\operatorname{cosec} t\cot t\right)dt$$

$$=\int_x^{\pi/2}\left(f(t)\operatorname{cosec} t\right)' dt$$

$$=f\left(\frac{\pi}{2}\right)\operatorname{cosec}\left(\frac{\pi}{2}\right)-\frac{f(x)}{\sin x}=3-\frac{f(x)}{\sin x}$$

$$\therefore \lim_{x\to 0} g(x)=3-\lim_{x\to 0}\frac{f(x)}{\sin x}; \quad \text{as } f'(0) = 1$$

$$\Rightarrow \lim_{x\to 0} g(x)=3-1=2$$

MATCHING TYPE

Column 1, 2 and 3 contain conics, equation of tangents to the conics and points of contact, respectively.

Column – 1	Column – 2	Column – 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2},\frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y=mx+a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}},\frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y=mx+\sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}},\frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y=mx+\sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}},\frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3},\frac{1}{2}\right)$ is found to be $\sqrt{3}x+2y=4,$ then which of the following options is the only CORRECT combination ?

(A) (II) (iii) (R) (B) (IV) (iv) (S) (C) (IV) (iii) (S) (D) (II) (iv) (R)

Ans. (D)

$P\left(\sqrt{3},\frac{1}{2}\right)$: tangent $\sqrt{3}x+2y=4$

$\Rightarrow \left(\sqrt{3}\right)x+4\left(\frac{1}{2}\right)y=4$ comparing with (II)

$\Rightarrow a=2 \;\therefore\; y=mx+\sqrt{a^2m^2+1}$ is tangent for $m=-\frac{\sqrt{3}}{2}$ i.e.(ii)

$\therefore$ point of contact for a = 2, $m=-\frac{\sqrt{3}}{2}$ is R

50. If a tangent to a suitable conic (Column 1) is found to be y = x + 8 and its point of contact is (8,16), then which of the following options is the only CORRECT combination ?

(A) (III) (i) (P) (B) (III) (ii) (Q) (C) (II) (iv) (R) (D) (I) (ii) (Q)

Ans. (A)

y = x + 8 is tangent ⇒ m = 1; P(8, 16)

Comparing tangent with (i) of column 2, m = 1 satisfied and a = 8 obtained which matches for point of contact (P) of column 3 and (III) of column I.

51. For $a=\sqrt{2},$ if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1,1), then which of the following options is the only CORRECT combination for obtaining its equation ?

(A) (II) (ii) (Q) (B) (III) (i) (P) (C) (I) (i) (P) (D) (I) (ii) (Q)

Ans. (D)

For $a=\sqrt{2}$ and point (–1,1) only I of column-1 satisfies. Hence equaiton of tangent is –x + y = 2 or y = x + 2 $\Rightarrow$ m = 1 which matches with (ii) of column 2 and also with Q of column 3

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0, \infty)$.

MATCHING TYPE

* Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
* Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
* Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column – 1	Column – 2	Column – 3
(I) $f(x) = 0$ for some $x \in (1,e^2)$	(i) $\lim_{x\to\infty} f(x)=0$	(P) f is increasing in (0,1)
(II) $f'(x) = 0$ for some $x \in (1,e)$	(ii) $\lim_{x\to\infty} f(x)=-\infty$	(Q) f is decreasing in (e,e^2)
(III) $f'(x) = 0$ for some $x \in (0,1)$	(iii) $\lim_{x\to\infty} f'(x)=-\infty$	(R) f' is increasing in (0,1)
(IV) $f''(x) = 0$ for some $x \in (1,e)$	(iv) $\lim_{x\to\infty} f''(x)=0$	(S) f' is decreasing in (e,e^2)

52. Which of the following options is the only CORRECT combination ?
(A) (IV) (i) (S) (B) (I) (ii) (R) (C) (III) (iv) (P) (D) (II) (iii) (S)

Ans. (D)

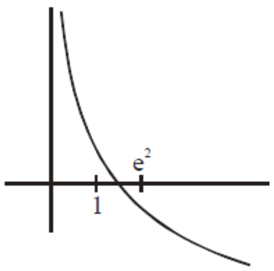
53. Which of the following options is the only CORRECT combination ?
(A) (III) (iii) (R) (B) (I) (i) (P) (C) (IV) (iv) (S) (D) (II) (ii) (Q)

Ans. (D)

54. Which of the following options is the only INCORRECT combination ?
(A) (II) (iii) (P) (B) (II) (iv) (Q) (C) (I) (iii) (P) (D) (III) (i) (R)

Ans. (D)

<u>Sol.(52 – 54)</u>



$$f(x)=x+\ell nx-x\ell nx, x>0$$

$$f'(x)=\not{1}+\frac{1}{x}-\ell nx-\not{1}$$

$$f''(x)=-\frac{1}{x^2}-\frac{1}{x}=\frac{-(x+1)}{x^2}$$

(I) $f(1)\, f(e^2) < 0$ so true
(II) $f'(1)\, f'(e) < 0$ so true
(III) Graph of $f'(x)$ so (III) is false
(IV) Is false

As $\lim_{x\to 0} f(x)=\lim_{x\to\infty} x\left[1+\frac{\ell nx}{x}-\ell nx\right]=-\infty$

$\therefore$ (i) is false (ii) is true

$\lim_{x\to\infty} f'(x)=-\infty$ so (iii) is true

$\lim_{x\to\infty} f''(x)=0$ so (iv) is true.

(P) $f'(x)$ is positive in (0,1) so true
(Q) $f'(x) < 0$ for in (e,e^2) so true
As $f'(x) < 0 \ \forall\ x > 0$ therefor R is false, S is true.
Alternate :
$f(x) = x + \ell nx - x\ell nx$

$f'(x) = \frac{1}{x} - \ell nx = 0$ at x = x_0 where $x_0 \in (1, e)$

$f''(x) = -\frac{1}{x^2} - \frac{1}{x} < 0 \forall x > 0 \Rightarrow f(x)$ concave down