JEE ADVANCED-2017 PAPER-2

PHYSICS

ONLY ONE

1. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2 \text{ km s}^{-1}$. The minimum initial velocity (v_S) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)

(A) $v_S = 22 \text{ km s}^{-1}$ (B) $v_S = 72 \text{ km s}^{-1}$ (C) $v_S = 42 \text{ km s}^{-1}$ (D) $v_S = 62 \text{ km s}^{-1}$

Ans. (C)

Given $v_e = 11.2 \text{ km/sec} = \sqrt{\frac{2GM_e}{R_e}}$

From energy conservation

$\frac{1}{2}mv_s^2 - \frac{GM_sm}{r} - \frac{GM_em}{R_e} = 0 - 0$ where r = distance of rocket from Sun

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

given : $M_S = 3 \times 10^5 M_e$ & $r = 2.5 \times 10^4 R_e$

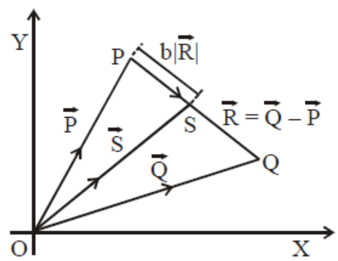
$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G3\times10^5 M_e}{2.5\times10^4 R_e}}$$

$$= \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3\times10^5}{2.5\times10^4}\right)}$$

$$= \sqrt{\frac{2GM_e}{R_e}\times13}$$

$$\Rightarrow v_s \approx 42 \text{ km/s}$$

2. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is



(A) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$ (C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (D) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

Ans. (C)

Let vector from point P to point S be $\vec{C}$

$$\Rightarrow \vec{C} = b|\vec{R}|\hat{R} = b|\vec{R}|\left(\frac{\vec{R}}{|\vec{R}|}\right) = b\vec{R} = b(\vec{Q} - \vec{P})$$

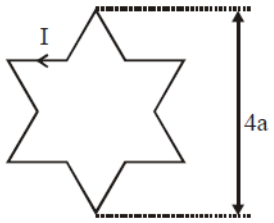
from triangle rule of vector addition

$\vec{P} + \vec{C} = \vec{S}$

$\vec{P} + b(\vec{Q} - \vec{P}) = \vec{S}$

$\Rightarrow \vec{S} = (1-b)\vec{P} + b\vec{Q}$

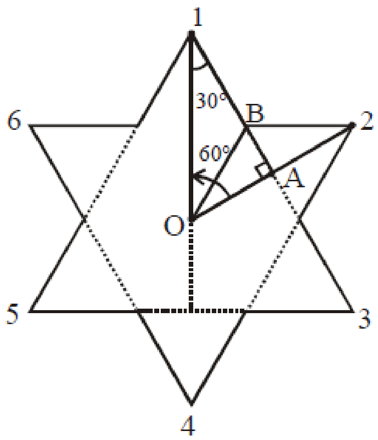
3. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is

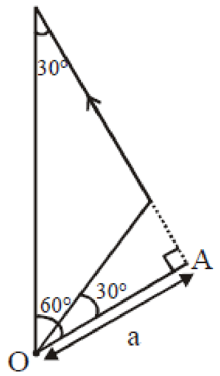


(A) $\frac{\mu_0 I}{4\pi a}3\left[\sqrt{3}-1\right]$ (B) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}-1\right]$ (C) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}+1\right]$ (D) $\frac{\mu_0 I}{4\pi a}3\left[2-\sqrt{3}\right]$

Ans. (B)

The given points (1, 2, 3, 4, 5, 6) makes 360° angle at 'O'. Hence angle made by vertices 1 & 2 with 'O' is 60°.





Direction of magnetic field at 'O' due to each segment is same. Since it is symmetric star shape, magnitude will also be same.

Magnetic field due to section BC.

$$(B_1) = \frac{ki}{a}\left(\sin(+60) - \sin 30\right) = \frac{ki}{2a}\left(\sqrt{3}-1\right)$$

$$B_{net} = 12 \times B_1 = \frac{6ki}{a}\left(\sqrt{3}-1\right) \& \left(k = \frac{\mu_0}{4\pi}\right)$$

4. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$.

The fastest photoelectron has a de-Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(A) λ_d^3/λ^2 (B) λ_d^3/λ (C) λ_d^2/λ^2 (D) λ_d/λ

Ans. (A)

According to photo electric effect equation :

$$KE_{max} = \frac{hc}{\lambda} - \phi_0$$

$$\frac{p^2}{2m} = \frac{hc}{\lambda} - \phi_0$$

$$\frac{(h/\lambda_d)^2}{2m} = \frac{hc}{\lambda} - \phi_0$$

Assuming small changes, differentiating both sides,

$$\frac{h^2}{2m}\left(-\frac{2d\lambda_d}{\lambda_d^3}\right)=-\frac{hc}{\lambda^2}d\lambda$$

$$\frac{d\lambda_d}{d\lambda}\propto\frac{\lambda_d^3}{\lambda^2}$$

5. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is δT = 0.01 second and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity g = 10 ms^{-2} and the velocity of sound is 300 ms^{-1}. Then the fractional error in the measurement, $\delta L/L$, is closest to

(A) 0.2 % (B) 5 % (C) 3 % (D) 1 %

Ans. (D)

Total time taken

$$T=\sqrt{\frac{2L}{g}}+\frac{L}{c}$$

Now, for an error δL in L,

We have an error δT in T

So, $T+\delta T=\sqrt{\frac{2(L+\delta L)}{g}}+\frac{(L+\delta L)}{c}$

$$=\sqrt{\frac{2L}{g}\left(1+\frac{\delta L}{L}\right)}+\frac{L}{c}\left(1+\frac{\delta L}{L}\right)$$

Since, $\frac{\delta T}{T}$ is very small, hence

$\frac{\delta L}{L}$ is also small, so taking binomial approximation

$$T+\delta T=\sqrt{\frac{2L}{g}}\left(1+\frac{1}{2}\frac{\delta L}{L}\right)+\frac{L}{c}\left(1+\frac{\delta L}{L}\right)$$

$$T+\delta T=\left(\sqrt{\frac{2L}{g}}\right)+\sqrt{\frac{2L}{g}}\left(\frac{1}{2}\frac{\delta L}{L}\right)+\left(\frac{L}{c}\right)+\frac{L}{c}\left(\frac{\delta L}{L}\right)$$

$$=\left(\sqrt{\frac{2L}{g}}+\frac{L}{c}\right)+\left(\frac{1}{2}\sqrt{\frac{2L}{g}}+\frac{L}{c}\right)\left(\frac{\delta L}{L}\right)$$

$$=T+\left(\frac{1}{2}\sqrt{\frac{2\times 20}{10}}+\frac{20}{300}\right)\frac{\delta L}{L}$$

$$\Rightarrow \delta T=\left(1+\frac{1}{15}\right)\frac{\delta L}{L}$$

$$\Rightarrow \frac{\delta L}{L}=\left(\frac{15}{16}\right)\delta T$$

$$=\left(\frac{15}{16}\right)\left(\frac{1}{100}\right)$$

% error $=\left(\frac{\delta L}{L}\right)\times 100\%$

$$=\frac{15}{16}\%$$

$\approx$ 1%

6. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate

of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point of the surface of the expanding sphere is proportional to

(A) R^3 (B) $\frac{1}{R}$ (C) R (D) $R^{2/3}$

Ans. (C)

Density of sphere is $\rho = \frac{m}{v} = \frac{3m}{4\pi R^3}$

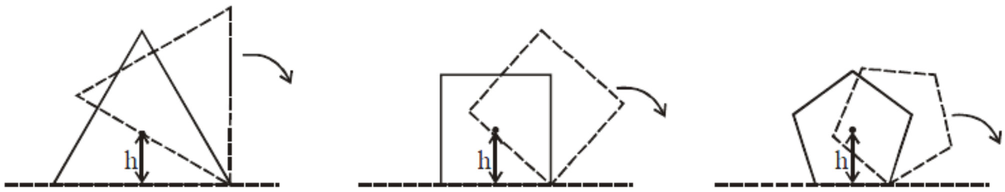
$\Rightarrow \frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{R}\frac{dR}{dt}$

Since $\Rightarrow \frac{1}{\rho}\frac{d\rho}{dt}$ is constant

$\therefore \frac{dR}{dt} \propto R$

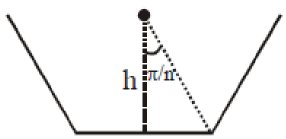
Velocity of any point on the circumfrence V is equal to $\frac{dR}{dt}$ (rate of change of radius of outer layer).

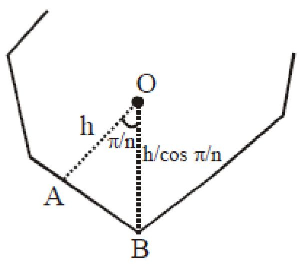
7. Consider regular polygons with number of sides n = 3, 4, 5 ..... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as



(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (B) $\Delta = h\sin^2\left(\frac{2\pi}{n}\right)$ (C) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$ (D) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

Ans. (C)





OA = h

$OB = \frac{h}{\cos\frac{\pi}{n}}$

Initial height of COM = h

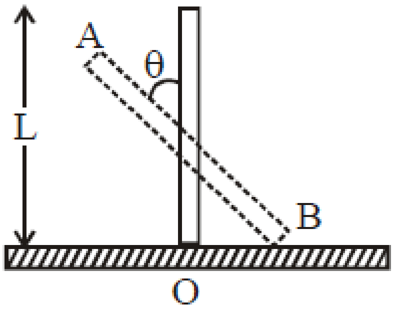
Final height of $COM = \frac{h}{\cos\left(\frac{\pi}{n}\right)}$

$\therefore \Delta = \frac{h}{\cos\frac{\pi}{n}} - h$

$$\Delta = h\left[\frac{1}{\cos\frac{\pi}{n}} - 1\right]$$

ONE OR MORE THAN ONE

8. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is □. Which of the following statements about its motion is/are correct ?



(A) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

(B) The midpoint of the bar will fall vertically downward

(C) Instantaneous torque about the point in contact with the floor is proportional to sin□

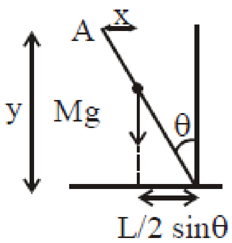
(D) The trajectory of the point A is a parabola

Ans. (ABC)

When the bar makes an angle θ; the height of its COM (mid point) is $\frac{L}{2}\cos\theta$

$\therefore$ displacement $= L - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$

Since force on COM is only along the vertical direction, hence COM is falling vertically downward. Instantaneous torque about point of contact is



$Mg \times \frac{L}{2}\sin\theta$

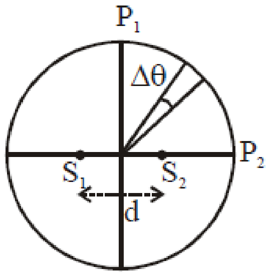
Now; $x = \frac{L}{2}\sin\theta$

$y = L\cos\theta$

$$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1$$

Path of A is an ellipse.

9. Two coherent monochromatic point sources S_1 and S_2 of wavelength λ = 600 nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct ?



(A) A dark spot will be formed at the point P_2
(B) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant
(C) At P_2 the order of the fringe will be maximum
(D) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

Ans. (CD)

At point P_2; $\Delta x = d = 1.8$ mm $= 3000\ \lambda$

hence a (bright fringe) will be formed at P_2

Now, $\Delta x = d \cos\theta = n\lambda$

$$\cos\theta = \frac{n\lambda}{d}$$

$$-\sin\theta\,\Delta\theta = (\Delta n)\frac{\lambda}{d}$$

$$\Delta\theta = -(\Delta n)\frac{\lambda}{d\sin\theta}$$

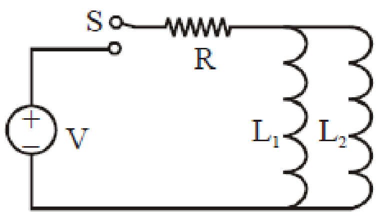
$\Delta\theta$ increases as θ decreases

At P_2, the order of fringe will be maximum.

For total no. of bright fringes $d = n\lambda \Rightarrow n = 3000$

$\therefore$ total no. of fringes = 3000

10. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct?



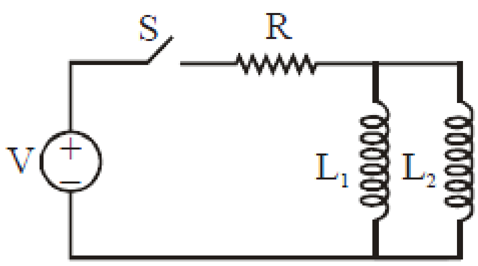
(A) The ratio of the currents through L_1 and L_2 is fixed at all times (t > 0)

(B) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(C) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(D) At t = 0, the current through the resistance R is $\frac{V}{R}$

Ans. (ABC)



Since inductors are connected in parallel

$V_{L_1} = V_{L_2}$

$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt}$

$L_1 I_1 = L_1 I_2$

$\frac{I_1}{I_2} = \frac{L_2}{L_1}$

Current through resistor at any time t is given by

$I = V/R\left(1 - e^{-\frac{RT}{L}}\right)$ where $L = \frac{L_1 L_2}{L_1 + L_2}$

After long time $I = \frac{V}{R}$

$I_1 + I_2 = I$ ...(i)

$L_1 I_1 = L_2 I_2$ ...(ii)

From (i) & (ii) we get

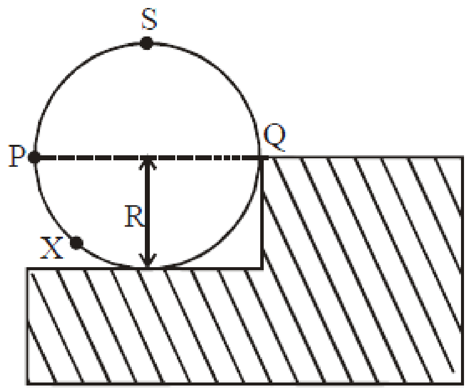
$I_1 = \frac{V}{R}\frac{L_2}{L_1 + L_2}, \quad I_2 = \frac{V}{R}\frac{L_1}{L_1 + L_2}$

(D) value of current is zero at t = 0

value of current is V/R at t = ∞

Hence option (D) is incorrect.

11. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct ?



(A) If the force is applied normal to the circumference at point X then τ is constant

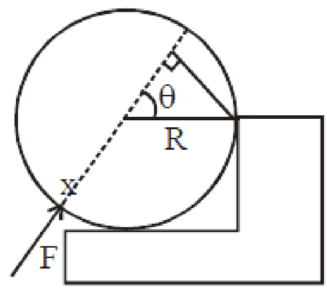
(B) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

(C) If the force is applied normal to the circumference at point P then τ is zero

(D) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

Ans. (C)

(A) is incorrect

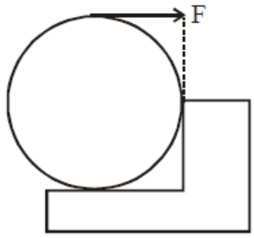


If force is applied normal to surface at point X

$\tau = F_y R \sin\theta$

Thus τ depends on θ & it is not constant

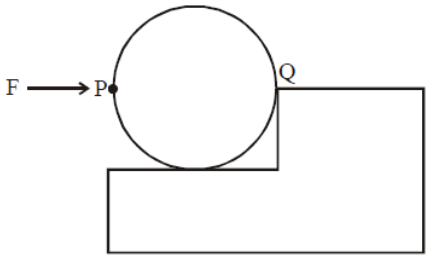
(B) is incorrect



if force applied tangentially at S

$\tau = F \times R \neq 0$

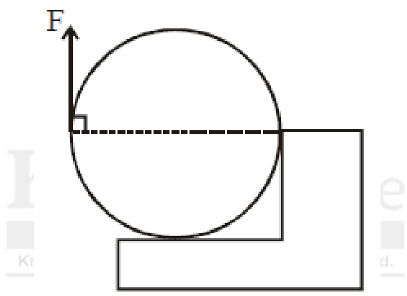
but it will climb as mentioned in question.



If force is applied normal to surface at P then line of action of force will pass from Q & thus

$\tau = 0$

(D) is incorrect.



if force is applied at P tangentially the

$\tau = F \times 2R$ = constant

12. The instantaneous voltages at three terminals marked X, Y and Z are given by

$V_X = V_0 \sin \omega t$

$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and $V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be:-

(A) $V_{XY}^{rms} = V_0$ (B) $V_{YX}^{rms} = V_0\sqrt{\frac{1}{2}}$

(C) Independent of the choice of the two terminals

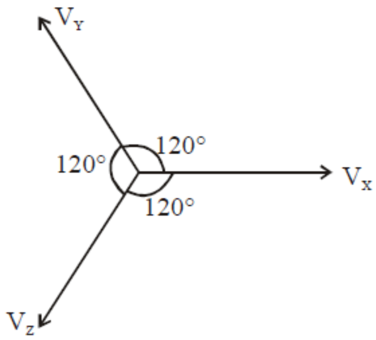
(D) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

Ans. (CD)

Potential difference between X & Y = $V_X - V_Y$

Potential difference between Y & Z = $V_Y - V_Z$

Phasor of the voltages :



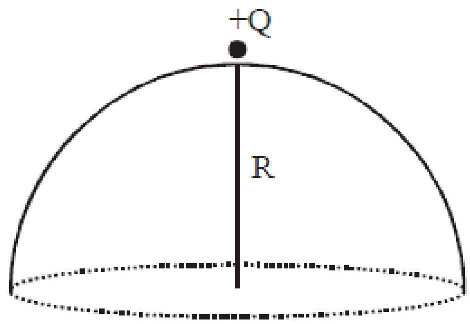
$\therefore\ V_X - V_Y = \sqrt{3}V_0$

$V_{XY}^{rms} = \dfrac{\sqrt{3}V_0}{\sqrt{2}}$

similarly $V_{YZ}^{rms} = \dfrac{\sqrt{3}V_0}{\sqrt{2}}$

Also difference is independent of choice of two terminals.

13. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct ?



(A) The circumference of the flat surface is an equipotential

(B) The electric flux passing through the curved surface of the hemisphere is $-\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

(C) Total flux through the curved and the flat surfaces is $\dfrac{Q}{\varepsilon_0}$

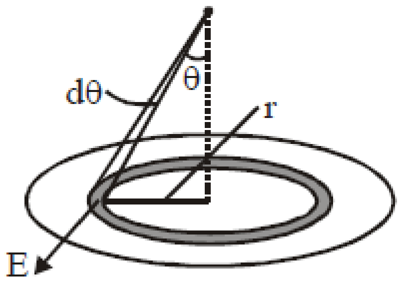
(D) The component of the electric field normal to the flat surface is constant over the surface

Ans. (AB)

Every point on circumference of flat surface is at equal distance from point charge

Hence circumference is equipotential.

Flux passing through curved surface = – flux passing through flat surface.



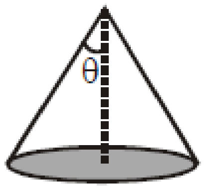
$$(d\phi)_{\text{through the ring}} = E\cos\theta.dA = \frac{1}{4\pi\in_0}\frac{Q}{\left(\sqrt{r^2+R^2}\right)^2}\frac{R}{\sqrt{R^2+r^2}}.2\pi rdr$$

$$\therefore \int d\phi = \frac{QR}{4\pi\in_0}2\pi\int_0^R \frac{rdr}{\left(R^2+r^2\right)^{3/2}} = \frac{q}{2\in_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

$\therefore$ Flux through curved surface $= -\dfrac{q}{2\epsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

Note : Flux through surface can be calculated using concept of solid angle.

$\Omega = 2\pi(1-\cos\theta) = 2\pi\left(1-\dfrac{1}{\sqrt{2}}\right)$



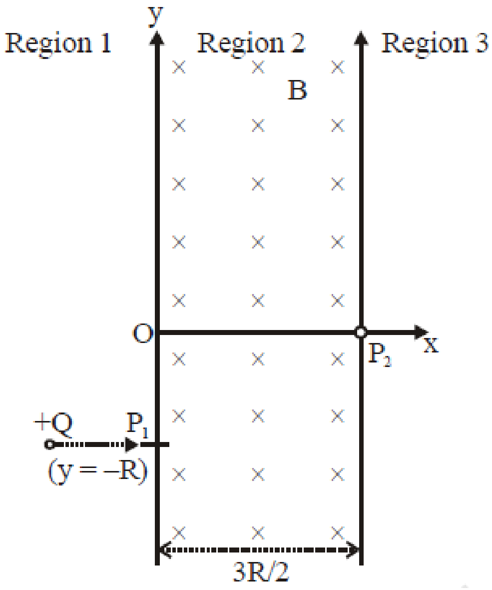
$\therefore$ Solid angle subtended $= 2\pi\left(1-\dfrac{1}{\sqrt{2}}\right)$

ϕ for 4π solid angle $= \dfrac{q}{\epsilon_0}$

$\therefore$ ϕ for $2\pi\left(1-\dfrac{1}{\sqrt{2}}\right)$ solid angle $= \dfrac{q}{4\pi\epsilon_0}.2\pi\left(1-\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{q}{2\epsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

14. A uniform magnetic field B exists in the region between x = 0 and x = $\dfrac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point P_1(y = –R). Which of the following options(s) is/are correct ?



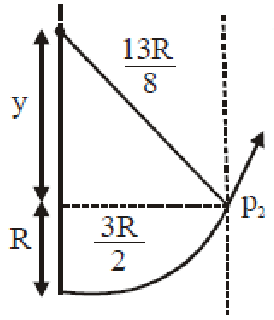
(A) For $B = \dfrac{8}{13}\dfrac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

(B) For $B > \dfrac{2}{3}\dfrac{p}{QR}$, the particle will re-enter region 1

(C) For a fixed B, particle of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

(D) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the chage in its linear momentum between point P_1 and the farthest point from y-axis is $\dfrac{p}{\sqrt{2}}$

Ans. (AB)



For $B = \frac{8}{13}\frac{p}{QR}$, radius of path

$$R' = \frac{p}{Q.B} = \frac{p \times 13QR}{Q8p} = \frac{13}{8}R$$

using pythagorous theorem, $y = \frac{5R}{8}$

$\therefore$ particle will enter region 3 through point P_2

for $B > \frac{2}{3}\frac{p}{QR}$

Radius of path $< \frac{3PQR}{Q2p} = \frac{3}{2}R$

$\therefore$ Particle will not enter in region 3 & will re-enter region 1 charge in momentum $= \sqrt{2}\,p$. When particle enters region 1 between entry point & farthest point from y-axis.

PARAGRAPH–1(15 & 16)

Consider a simple RC circuit as shown in figure 1.

Process 1 : In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2 : In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

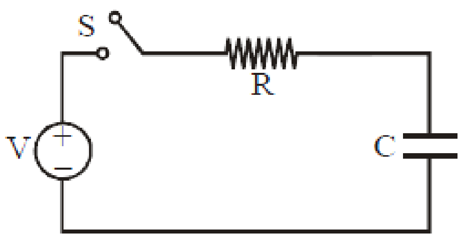


Figure 1

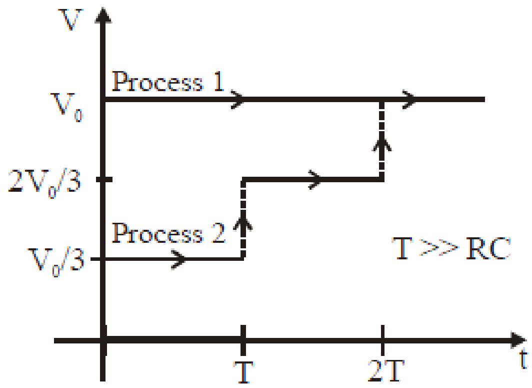
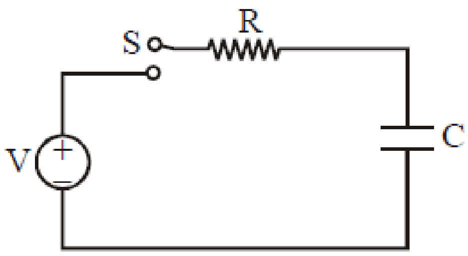


Figure 2

15. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by

(A) $E_C = E_D$ (B) $E_C = 2E_D$ (C) $E_C = \frac{1}{2}E_D$ (D) $E_C = E_D \ln 2$

Ans. (A)



When switch is closed for a very long time capacitor will get fully charged & charge on capacitor will be q = CV

Energy stored in capacitor $\in_C = \frac{1}{2}CV^2$ .....(i)

Work done by battery (ω) = Vq = VCV = CV^2

dissipated across resistance $\in_D$ = (work done by battery) – (energy store)

$\in_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2$ .....(ii)

from (i) and (ii)

$\in_D = \in_C$

16. In Process 2, total energy dissipated across the resistance E_D is

(A) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (C) $E_D = \frac{1}{2}CV_0^2$ (D) $E_D = 3CV_0^2$

Ans. (A)

For process (1)

Charge on capacitor $= \frac{CV_0}{3}$

energy stored in capacitor $= \frac{1}{2}C\frac{V_0^2}{9} = \frac{CV_0^2}{18}$

work done by battery $= \frac{CV_0}{3} \times \frac{V}{3} = \frac{CV_0^2}{9}$

Heat loss $= \frac{CV_0^2}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$

For process (2)

Charge on capacitor $= \frac{2CV_0}{3}$

Extra charge flow through battery $= \frac{CV_0}{3}$

Work done by battery : $\frac{CV_0}{3} . \frac{2V_0}{3} = \frac{2CV_0^2}{9}$

Final energy store in capacitor : $\frac{1}{2}C\left(\frac{2V_0}{3}\right)^2 = \frac{4CV_0^2}{18}$

energy store in process 2 : $\frac{4CV_0^2}{18} - \frac{CV_0^2}{18} = \frac{3CV_0^2}{18}$

Heat loss in process (2) = work done by battery in process (2) – energy store in capacitor process (2)

$= \frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$

For process (3)

Charge on capacitor = CV_0

extra charge flow through battery : $CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$

work done by battery in this process : $\left(\frac{CV_0}{3}\right)(V_0) = \frac{CV_0^2}{3}$

find energy store in capacitor : $\frac{1}{2}CV_0^2$

energy stored in this process : $\frac{1}{2}CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$

heat loss in process (3) : $\frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$

Now total heat loss (E_D) : $\frac{CV_0^2}{18} + \frac{CV_0^2}{18} + \frac{CV_0^2}{18} = \frac{CV_0^2}{6}$

final energy store in capacitor : $\frac{1}{2}CV_0^2$

so we can say that $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

PARAGRAPH–2(17 & 18)

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

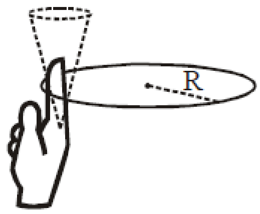


Figure 1

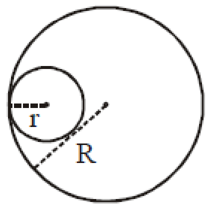


Figure 2

17. The total kinetic energy of the ring is

(A) $M\omega_0^2R^2$ (B) $M\omega_0^2(R-r)^2$ (C) $\frac{1}{2}M\omega_0^2(R-r)^2$ (D) $\frac{3}{2}M\omega_0^2(R-r)^2$

Ans. (B)

18. The minimum value of ω_0 below which the ring will drop down is

(A) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (B) $\sqrt{\frac{g}{\mu(R-r)}}$ (C) $\sqrt{\frac{2g}{\mu(R-r)}}$ (D) $\sqrt{\frac{2g}{2\mu(R-r)}}$

Ans. (B)

ONLY ONE

19. Which of the following combination will produce H_2 gas ?

(A) Zn metal and NaOH(aq)
(B) Au metal and NaCN(aq) in the presence of air
(C) Cu metal and conc. HNO_3
(D) Fe metal and conc. HNO_3

Ans. (A)

(A) $Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2$

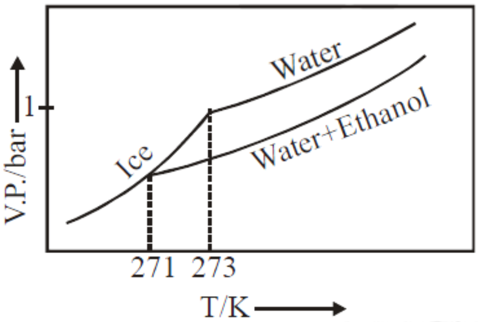
(B) $4Au + 8NaCN + O_2 + 2H_2O \longrightarrow 4Na[Au(CN)_2] + 4NaOH$

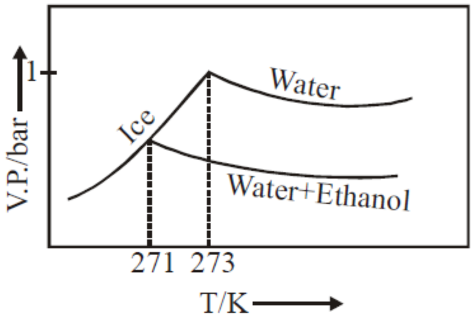
(C) $Cu + \underset{(conc.)}{4HNO_3} \longrightarrow Cu(NO_3)_2 + 2NO_2 + 2H_2O$

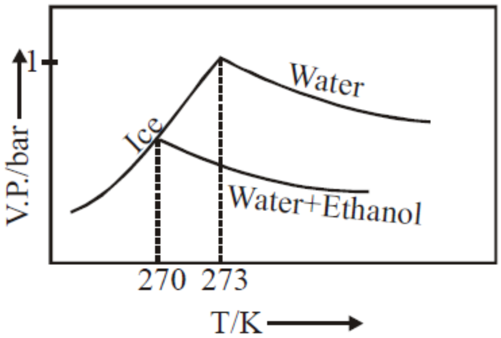
(D) Formation of passive layer of Fe_2O_3 on the surface of Fe and NO_2 gas is evolved.

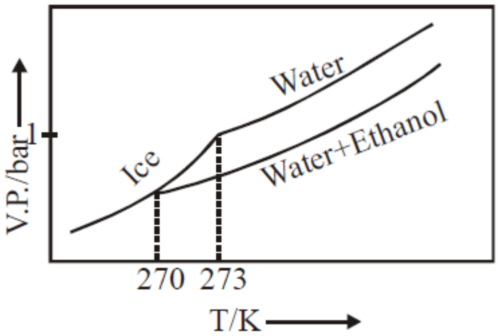
20. Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg mol^{-1}. The figures shown below represents plots of vapour pressure (V.P.) versus temperature (T). [Molecular weight of ethanol is 46 g mol–1]

Among the following, the option representing change in the freezing point is

(A) 

(B) 

(C) 

(D) 

Ans. (D)

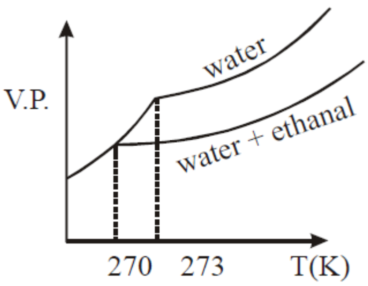
Ethanol should be considered non volatile as per given option

$\Delta T_f = K_f \times m$

$$\Delta T_f = 2 \times \frac{34.5}{46.05} = 3K$$

$T_f^0 = 273K$

$T_f = 270$ K



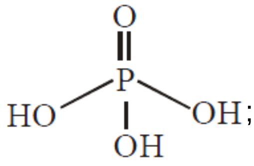
21. The order of the oxidation state of the phosphorus atom in H_3PO_2, H_3PO_4, H_3PO_3 and $H_4P_2O_6$ is

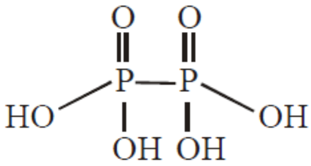
(A) $H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

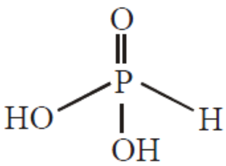
(B) $H_3PO_3 > H_3PO_2 > H_3PO_4 > H_4P_2O_6$

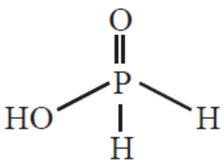
(C) $H_3PO_2 > H_3PO_3 > H_4P_2O_6 > H_3PO_4$

(D) $H_3PO_4 > H_3PO_2 > H_3PO_3 > H_4P_2O_6$

$H_3PO_4 \rightarrow$  ; oxidation state of P = + 5

$H_4P_2O_6 \rightarrow$  ; oxidation state of P = + 4

$H_3PO_4 \rightarrow$  ; oxidation state of P = +3

$H_3PO_2 \rightarrow$  ; oxidation state of P = +1

Hence Ans (A)

22. The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at T = 298 K are

$\Delta_f G°$ [C(graphite)] = 0 kJ mol^{-1}

$\Delta_f G°$ [C(diamond)] = 2.9 kJ mol^{-1}

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by 2×10^{-6} m^3 mol^{-1}. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with C(diamond), is

[Useful information : 1 J = 1 kg m^2 s^{-2}; 1 Pa = 1 kg m^{-1} s^{-2}; 1 bar = 105 Pa]

(A) 14501 bar (B) 29001 bar (C) 58001 bar (D) 1405 bar

Ans. (A)

C(graphite) → C(diamond) ; $\Delta G^0 = \Delta_f G^0_{diamond} - \Delta_f G^0_{graphite}$ = 2.9 kJ/mole at 1 bar

As $dG_T = V.dP$

$$\int_{\Delta G_1}^{\Delta G_2} d(\Delta G_T) = \int_{P_1}^{P_2} \Delta V.dP$$

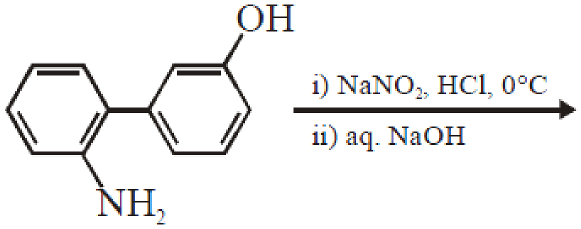
$\Delta G_2 - \Delta G_1 = \Delta V.(P_2 - P_1)$

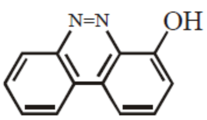
$(2.9 \times 10^3 - 0) = (-2 \times 10^{-6})(1 - P_2)$

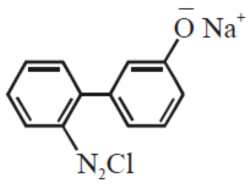
$$P_2 - 1 = \frac{2.9 \times 10^3}{2 \times 10^{-6}} Pa = 1.45 \times 10^4 bar$$

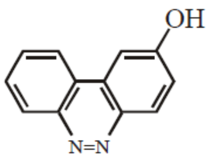
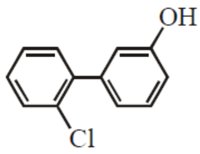
P_2 = 14501 bar

23. The major product of the following reaction is

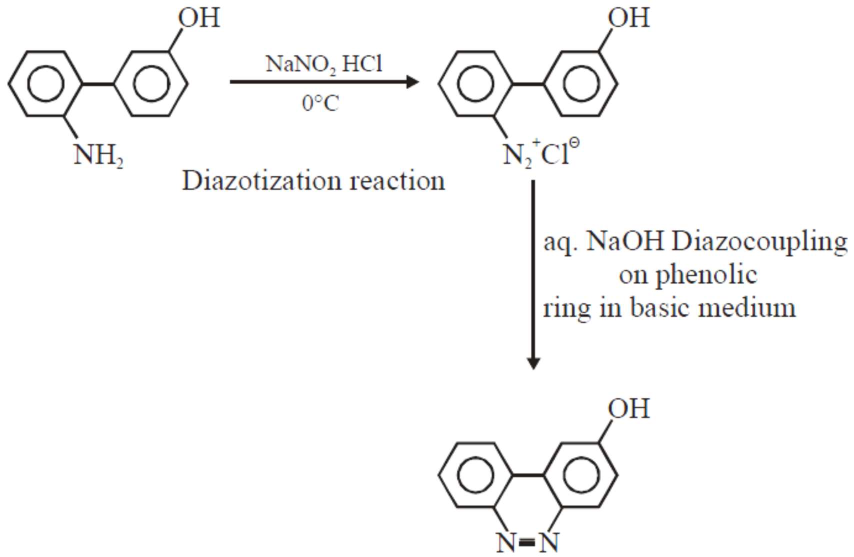


(A) 

(B) 

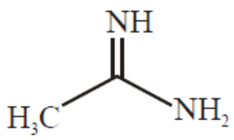
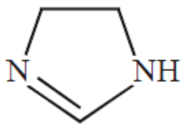
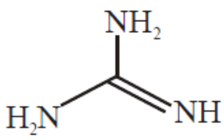
(C)  (D) 

Ans. (C)



Para is less hindered

24. The order of basicity among the following compounds is

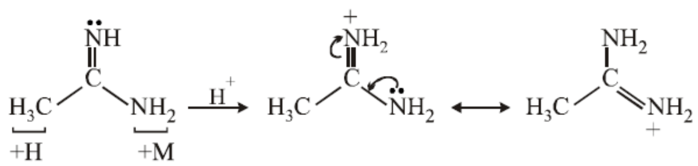
I II III IV

(A) II > I > IV > III (B) IV > II > III > I (C) I > IV > III > II (D) IV > I > II > III

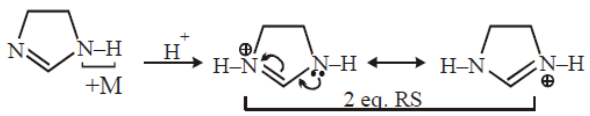
Ans. (D)

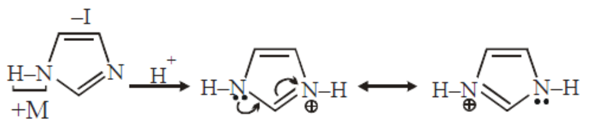
Basic strength $\propto$ stability of conjugated acid.

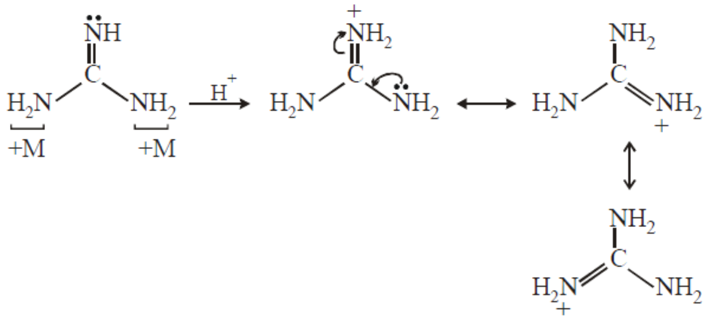
$\propto +M / +H / +I$

(I) 

Conjugated acid stabilized by 2 equivalent R.S.

(II) 

(III) 

(IV) 

Conjugated acid stabilized by 3 equivalent R.S.

25. For the following cell :

$Zn(s) \mid ZnSO_4\ (aq.) \parallel CuSO_4\ (aq.) \mid Cu(s)$

when the concentration of Zn^{2+} is 10 times the concentration of Cu^{2+}, the expression for ΔG (in J mol^{-1}) is

[F is Faraday constant , R is gas constant, T is temperature , E^o(cell) = 1.1V]

(A) 2.303 RT + 1.1F (B) 2.303 RT – 2.2F

(C) 1.1 F (D) –2.2 F

Ans. (B)

$\Delta G = \Delta G^0 + 2.303\ RT \log Q$

$\Delta G = -\ nFE^0 + 2.303\ RT \log Q$

sGiven : E^o = 1.1 V and n = 2

$$\Delta G = (-\ 2 \times 1.1 \times F) + 2.303\ RT \log \left[\frac{[Zn^{+2}]}{[Cu^{+2}]}\right]$$

$\Delta G = -2.2\ F + 2.303\ RT$

ONE OR MORE THAN ONE

26. In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are):

(A) The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

(B) The activation energy of the reaction is unaffected by the value of the steric factor

(C) Since P = 4.5, the reaction will not proceed unless an effective catalyst is used.

(D) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation.

Ans. (BD)

$K = P.A.\ e^{-Ea/RT}$

(A) If P < 1 $A_{arr.} > A_{expt}$

(D) If P > 1 $A_{arr.} < A_{expt}$

(C) If P is very small, then catalyst is required to carry out the reaction at measurable rate.

27. Compound **P** and **R** upon ozonolysis produce **Q** and **S**, respectively. The molecular formula of **Q** and **S** is C_8H_8O. **Q** undergoes Cannizzaro reaction but not haloform reaction, whereas **S** undergoes haloform reaction but not Cannizzaro reaction.

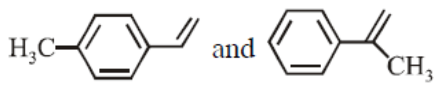
(i) $P \xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i) } O_3/CH_2/Cl_2} Q$

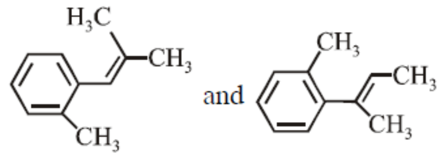
(C_8H_8O)

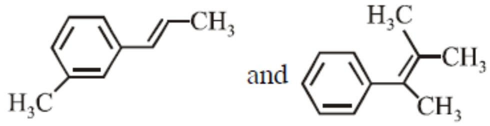
(ii) $R \xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i) } O_3/CH_2/Cl_2} S$

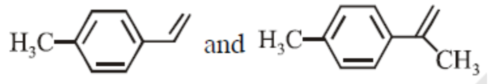
(C_8H_8O)

The option(s) with suitable combination of P and R, respectively, is(are)

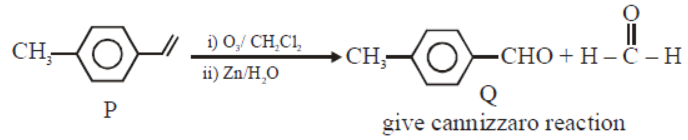
(A) 

(B) 

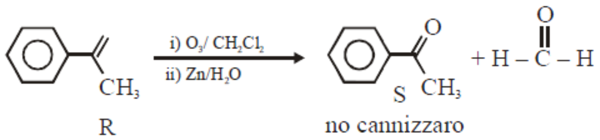
(C) 

(D) 

Ans. (AC)

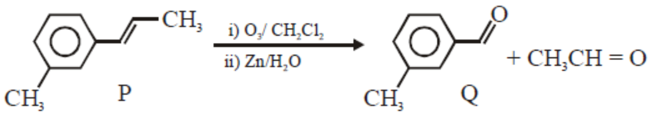
(A) 

give cannizzaro reaction
but no haloform

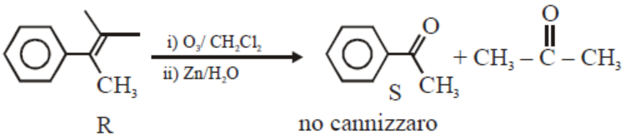


no cannizzaro
but give haloform

(B) Product of ozonolysis of R is having 9 carbon.

(C) 

give cannizzaro reaction
but no haloform



no cannizzaro
but give haloform

(D) Product of ozonolysis of R is having 9 carbon.

28. For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by

(A) With increase in temperature, the value of K for exothermic reaction decreases because the entropy change of the system is positive

(B) With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases

(C) With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surroundings decreases

(D) With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system negative

$$\Delta S_{surr.} = \frac{-q_{process}}{T_{surr.}}$$

If $\Delta H > 0$ on $T \uparrow K_{eq} \uparrow$, $\Delta S_{surr.} < 0$ (Surrounding is unfavourable)

If $\Delta H < 0$ on $T \uparrow K_{eq} \downarrow$, $\Delta S_{surr.} > 0$ (Surrounding is favourable)

29. The option(s) with only amphoteric oxides is (are):

(A) Cr_2O_3, CrO, SnO, PbO

(B) NO, B_2O_3, PbO, SnO_2

(C) Cr_2O_3, BeO, SnO, SnO_2

(D) ZnO, Al_2O_3, PbO, PbO_2

Ans. (CD)

(C) Cr_2O_3, BeO, SnO, SnO_2

all are amphoteric oxides

(D) ZnO, Al_2O_3, PbO, PbO_2

all are amphoteric oxides

30. Among the following, the correct statement(s) is are

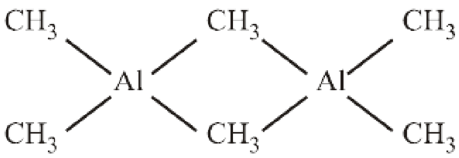
(A) $Al(CH_3)_3$ has the three-centre two-electron bonds in its dimeric structure

(B) $AlCl_3$ has the three-centre two-electron bonds in its dimeric structure

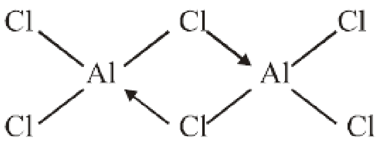
(C) BH_3 has the three-centre two-electron bonds in its dimeric structure

(D) The Lewis acidity of BCl_3 is greater than that of $AlCl_3$

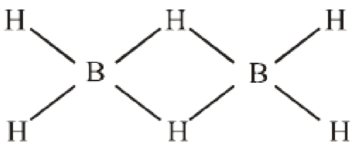
Ans. (ACD)

(A) 

Two 3c - $2e^-$ bond
(three-centre-two-electron)

(B) 

Two 3c - $4e^-$ bond
(three-centre-four-electron)

(C) 

Two 3c - $2e^-$ bond
(three-centre-two-electron)

(D) Lewis acidic strength decreases down the group. The decrease in acid strength occurs because as size increases, the attraction between the incoming electron pair and the nucleus weakens.

Hence Lewis acidic strength of BCl_3 is more than $AlCl_3$.

31. The correct statement(s) about surface properties is (are)

(A) Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium

(B) Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system.

(C) Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution.

(D) The critical temperatures of ethane and nitrogen and 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature.

Ans. (BD)

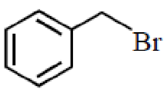
(A) Emulsion is liquid in liquid type colloid.

(B) For adsorption, $\Delta H < 0$ & $\Delta S < 0$

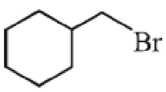
(C) Smaller the size and less viscous the dispersion medium, more will be the brownian motion.

(D) Higher the T_C, greater will be the extent of adsorption.

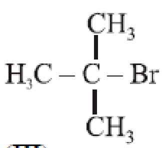
32. For the following compounds, the correct statement(s) with respect of nucleophilic substitution reactions is(are);



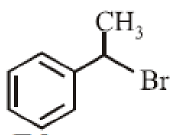
(I)



(II)



(III)



(IV)

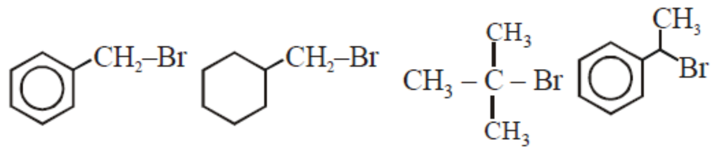
(A) I and II follow S_N2 mechanism

(B) The order of reactivity for I, III and IV is : IV > I > III

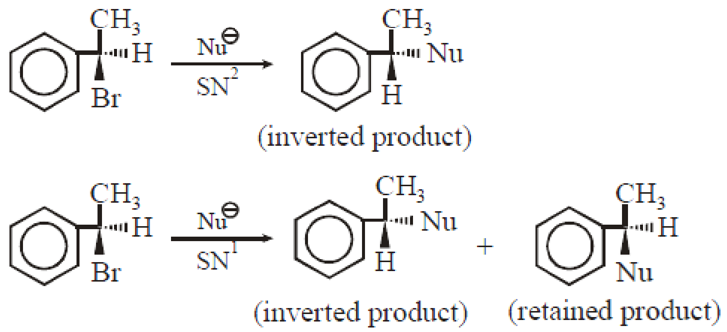
(C) I and III follow S_N1 mechanism

(D) Compound IV undergoes inversion of configuration

Ans. (ABCD)



(A) I and II follow S_N2 mechanism as they are primary
(B) Reactivity order IV > I > III
(C) I and III follows S_N1 mechanism as they form stable carbocation
(D) Compound IV undergoes inversion of configuration.



PASSAGE – I

Paragraph for Q.33 & 34

Upon heating $KClO_3$ in the presence of catalytic amount of MnO_2, a gas **W** is formed. Excess amount of **W** reacts with white phosphorus to give **X**. The reaction of **X** with pure HNO_3 gives **Y** and **Z**.

33. **W** and **X** are, respectively
(A) O_3 and P_4O_6 (B) O_2 and P_4O_{10} (C) O_3 and P_4O_{10} (D) O_2 and P_4O_6

34. **Y** and **Z** are, respectively
(A) N_2O_4 and H_3PO_3 (B) N_2O_4 and HPO_3
(C) N_2O_5 and HPO_3 (D) N_2O_3 and H_3PO_4

Sol. (33 & 34)

$$2KClO_3 \xrightarrow[\Delta]{MnO_2} 2KCl + \underset{(W)}{3O_2}$$

$$\underset{\text{white phosphorous}}{P_4} + \underset{\substack{(excess)\\(W)}}{5\,O_2} \longrightarrow \underset{(X)}{P_4O_{10}}$$

$$\underset{(X)}{P_4O_{10}} + 4\,HNO_3 \longrightarrow \underset{(Y)}{2N_2O_5} + \underset{(Z)}{(HPO_3)_4}$$

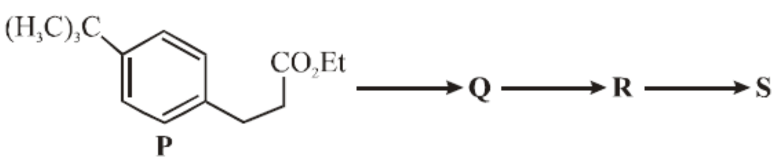
33. (B)
W and X are respectively
W = O_2 and X = P_4O_{10}

34. (C)
Y and Z are respectively
Y = N_2O_5 and Z = HPO_3

PASSAGE – II

Paragraph for Q.35 & 36

The reaction of compound P with CH_3MgBr (excess) in $(C_2H_5)_2O$ followed by addition of H_2O gives **Q**,
The compound **Q** on treatment with H_2SO_4 at 0ºC gives **R**. The reaction of **R** with CH_3COCl in the presence of anhydrous $AlCl_3$ in CH_2Cl_2 followed by treatment with H_2O produces compounds S.
[Et it compounds **P** is ethyl group]



35. The reactions, **Q** to **R** and **S** to **S**, are

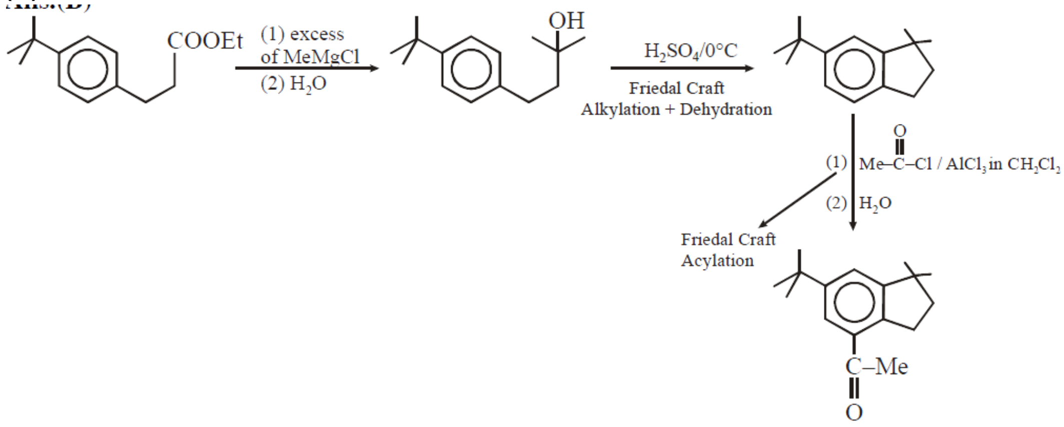
(A) Dehydration and Friedel -Crafts acylation

(B) Friedel-Crafts alkylation, dehydration and Friedel-Crafts acylation

(C) Aromatic sulfonation and Friedel-Crafts acylation

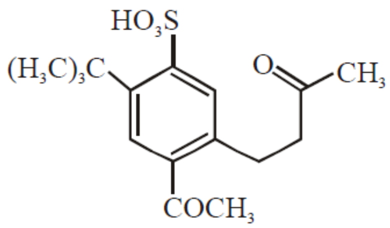
(D) Friedel-Crafts alkylation and Fridel-Crafts acylation

Ans. (B)

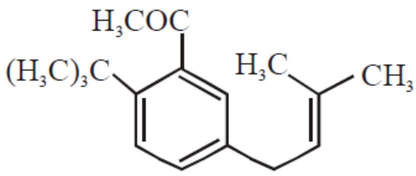


36. The product S is

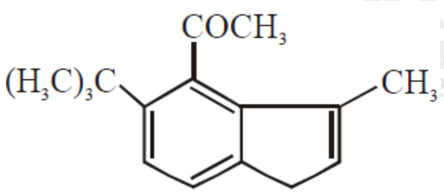
(A)



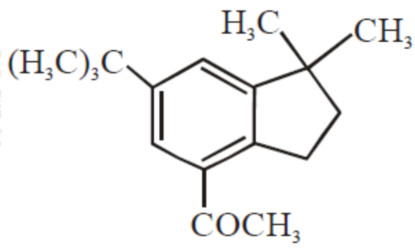
(B)



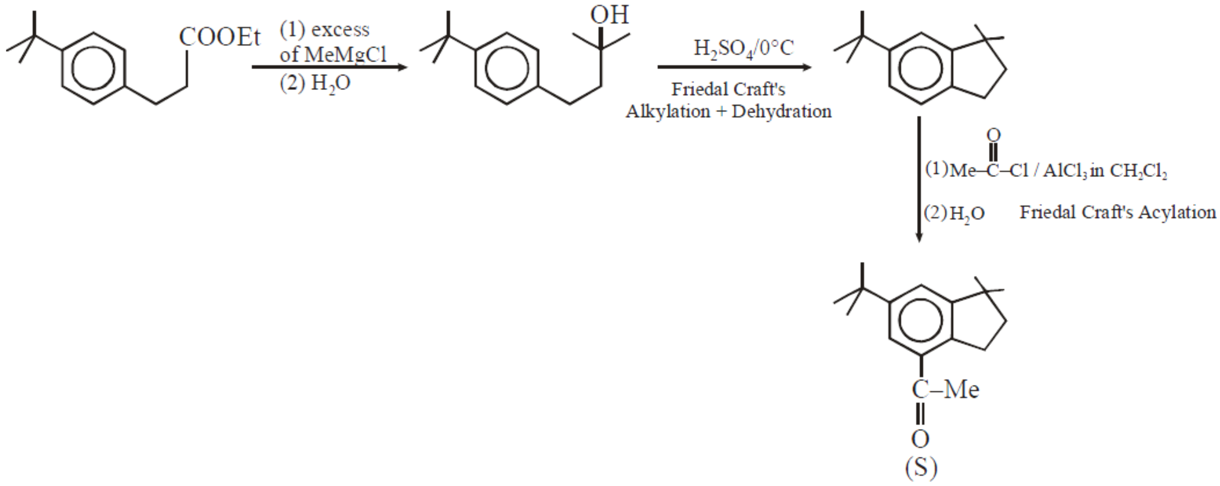
(C)



(D)



Ans. (D)



MATHEMATICS

ONLY ONE

37. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equationx + y + z = 10. Then the probability that z is even, is

(A) $\frac{36}{55}$ (B) $\frac{6}{11}$ (C) $\frac{5}{11}$ (D) $\frac{1}{2}$

Ans. (B)

Let z = 2k, where k = 0, 1, 2, 3, 4, 5

$\therefore$ x + y = 10 – 2k

Number of non negative integral solutions

$$\sum_{k=0}^{5} {}^{11-2k}C_1 = \sum 11 - 2k = 36$$

Total cases = ${}^{10+3-1}C_{3-1} = 66$

Reqd. prob. $= \frac{36}{66} = \frac{6}{11}$

38. Let S = {1, 2, 3,.....,9}. For k = 1,2, ..., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

(A) 125 (B) 252 (C) 210 (D) 126

Ans. (D)

$N_1 + N_2 + N_3 + N_4 + N_5$ = Total ways – {when no odd}

Total ways = ${}^9C_5$

Number of ways when no odd, is zero ($\because$ only available even are 2, 4, 6, 8)

$\therefore$ ${}^9C_5$ – zero = 126

39. If f : R → R is a twice differentiable function such that f''(x) > 0 for all x ∈R and $f\left(\frac{1}{2}\right) = \frac{1}{2}, f(1) = 1$, then

(A) $0 < f'(1) \le \frac{1}{2}$ (B) $f'(1) \le 0$ (C) $f'(1) > 1$ (D) $\frac{1}{2} < f'(1) \le 1$

Ans. (C)

Using LMVT on f(x) for $x \in \left[\frac{1}{2}, 1\right]$

$$\frac{f(1) - f\left(\frac{1}{2}\right)}{1 - \frac{1}{2}} = f'(c), \Rightarrow f'(c) = 1, \text{ where } c \in \left(\frac{1}{2}, 1\right)$$

$\because$ f'(x) is an increasing function $\forall$ x ∈ R

$\therefore$ $f'(1) > 1$

40. If y = y(x) satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx$, x > 0 and $y(0) = \sqrt{7}$, then y(256) =

(A) 80 (B) 3 (C) 16 (D) 9

Ans. (B)

$$y = \frac{1}{8}\int \frac{dx}{\sqrt{4+\sqrt{9+x}}.\sqrt{x}.\sqrt{9+\sqrt{x}}}$$

put $\sqrt{9+\sqrt{x}} = t \quad \Rightarrow \frac{dx}{\sqrt{x}.\sqrt{9+\sqrt{x}}} = 4\,dt$

$$\therefore \; y = \frac{4}{8}\int \frac{dt}{\sqrt{4+t}}$$

$$\Rightarrow y = \sqrt{4+t} + C$$

$\Rightarrow y(x)=\sqrt{4+\sqrt{9+\sqrt{x}}}+C$

at x = 0 : $y(0)=\sqrt{7} \;\; \Rightarrow \; C=0$

$\therefore \;\; y(x)=\sqrt{4+\sqrt{9+\sqrt{x}}}$

$\Rightarrow$ y(256) = 3

41. How many 3 × 3 matrices M with entries from {0,1,2} are there, for which the sum of the diagonal entries of M^TM is 5 ?

(A) 198 (B) 126 (C) 135 (D) 162

Ans. (A)

Let $M=\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$\therefore$ tr(M^TM) = $a^2 + b^2 + c^2 + d^2 + c^2 + f^2 + g^2 + h^2 + i^2 = 5$, where entries are {0,1,2}

Only two cases are possible.

(I) five entries 1 and other four zero

$\therefore \; {}^9C_5 \times 1$

(II) One entry is 2, one entry is 1 and others are 0.

$\therefore \; {}^9C_2 \times 2!$

Total = 126 + 72 = 198.

42. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

Then the triangle PQR has S as its

(A) incentre (B) orthocenter (C) circumcentre (D) centroid

Ans. (B)

Let position vector of $P(\vec{p}),Q(\vec{q}),R(\vec{r})\,\&\,S(\vec{r})$ with respect to $O(\vec{o})$

Now, $\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}$

$\Rightarrow \vec{p}.\vec{q}+\vec{r}.\vec{s}=\vec{r}.\vec{p}+\vec{q}.\vec{s}$

$\Rightarrow (\vec{p}-\vec{s}).(\vec{q}-\vec{r})=0$ .....(1)

Also, $\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

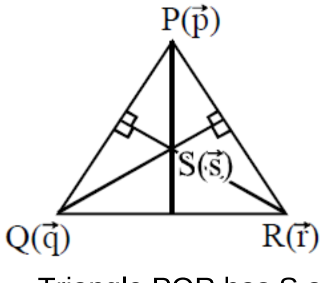
$\Rightarrow \vec{r}.\vec{p}+\vec{q}.\vec{s}=\vec{q}.\vec{r}+\vec{p}.\vec{s}$

$\Rightarrow (\vec{r}-\vec{s}).(\vec{p}-\vec{q})=0$ .....(2)

Also, $\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

$\Rightarrow \vec{p}.\vec{q}+\vec{r}.\vec{s}=\vec{q}.\vec{r}+\vec{p}.\vec{s}$

$\Rightarrow (\vec{q}-\vec{s}).(\vec{p}-\vec{r})=0$ .....(3)



$\Rightarrow$ Triangle PQR has S as its orthocentre

43. The equation of the plane passing through the point (1,1,1) and perpendicular to the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7, is-

(A) 14x + 2y + 15z = 31 (B) 14x + 2y – 15z = 1

(C) –14x + 2y + 15z = 3 (D) 14x – 2y + 15z = 27

Ans. (A)

The normal vector of required plane is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2\hat{j} - 15\hat{k}$$

$\therefore$ The equation of required plane passing through (1, 1, 1) will be

–14(x – 1) – 2(y – 1) – 15(z – 1) = 0

$\Rightarrow$ 14x + 2y + 15z = 31

ONE OR MORE THAN ONE

44. If $I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx$, then

(A) $I < \frac{49}{50}$ (B) $I < \log_e 99$ (C) $I > \frac{49}{50}$ (D) $I > \log_e 99$

Ans. (BC)

$$I = \sum_{k=1}^{98} \left(\int_{k}^{k+1} \frac{(k+1)}{x(x+1)} dx \right)$$

$$= \sum_{k=1}^{98} (k+1) \left(\int_{k}^{k+1} \left(\frac{1}{x} - \frac{1}{x+1} \right) dx \right)$$

$$= \sum_{k=1}^{98} (k+1) \left((\ell n x - \ell n(x+1))_{k}^{k+1} \right)$$

$$= \sum_{k=1}^{98} (k+1) \left(\ell n(k+1) - \ell n(k+2) - \ell n k + \ell n(k+1) \right)$$

$$= \sum_{k=1}^{98} \left((k+1)\ell n(k+1) - k.\ell n k \right) - \sum_{k=1}^{98} \left((k+1).\ell n(k+2) - k.\ell n(k+1) \right) + \sum_{k=1}^{98} \left(\ell n(k+1) - \ell n k \right)$$

(Difference series)

$$\therefore \quad I = (99\,\ell n 99) + (-99\,\ell n 100 + \ell n 2) + (\ell n 99) = \ell n \left(\frac{2 \times (99)^{100}}{(100)^{99}} \right) \qquad .....(1)$$

For option (B) :

Now, consider $(100)^{99} = (1 + 99)^{99}$

$$= {}^{99}C_0 + {}^{99}C_1(99) + {}^{99}C_2(99)^2 + ... + {}^{99}C_{97}(99)^{97} + \underbrace{{}^{99}C_{98}(99)^{98}}_{(\text{value} = (99)^{99})} + \underbrace{{}^{99}C_{99}(99)^{99}}_{(\text{value} = (99)^{99})}$$

$$\Rightarrow (100)^{99} > 2.(99)^{99} \Rightarrow \frac{2 \times (99)^{99}}{(100)^{99}} < 1$$

$$\therefore \quad \frac{2 \times (99)^{100}}{(100)^{99}} < 99 \quad \text{(on multiplying by 99)}$$

$$\Rightarrow I < \ell n 99$$

For option (C) :

Since, $\sum_{k=1}^{98} \int_{k}^{k+1} \frac{k+1}{(x+1)^2} dx < \sum_{k=1}^{98} \int_{k}^{k+1} \frac{(k+1)dx}{x(x+1)}$

$$\Rightarrow \sum_{k=1}^{98} \left(\frac{1}{k+2} \right) < I$$

(on integration)

$$\Rightarrow \underbrace{\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ... + \frac{1}{1000} \right)}_{98 \text{ terms}} < I$$

$$\Rightarrow \frac{98}{100} < \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots\ldots + \frac{1}{100} < I$$

$$\therefore \quad I > \frac{49}{50}$$

45. If f : R → R is a differentiable function such that $f'(x) > 2f(x)$ for all x ∈ R, and $f(0) = 1$, then
(A) $f(x) > e^{2x}$ in (0, ∞) (B) $f(x)$ is decreasing in (0,∞)
(C) $f(x)$ is increasing in (0, ∞) (D) $f'(x) < e^{2x}$ in (0,∞)

Ans. (AC)

Given that,

$f'(x) > 2f(x) \forall x \in R$

$\Rightarrow f'(x) - 2f(x) > 0 \ \forall x \in R$

$\therefore \ e^{-2x}\left(f'(x) - 2f(x)\right) > 0 \forall x \in R$

$\Rightarrow \frac{d}{dx}\left(e^{-2x}f(x)\right) > 0 \forall x \in R$

Let $g(x) = e^{-2x}f(x)$

Now, $g'(x) > 0 \ \forall \ x \in R$

$\Rightarrow$ g(x) is strictly increasing ∀ x ∈ R

Also, g(0) = 1

$\therefore \ \forall x > 0$

$\Rightarrow$ g(x) > g(0) = 1

$\therefore \ e^{-2x}.f(x) > 1 \ \forall x \in (0, \infty) \ \Rightarrow \ f(x) > e^{2x} \ \forall x \in (0, \infty)$

$\therefore$ option (A) is correct

As, $f'(x) > 2 f(x) > 2e^{2x} > 2 \ \forall \ x \in (0, \infty)$

$\Rightarrow$ f(x) is strictly increasing on x ∈ (0, ∞)

$\Rightarrow$ option (C) is correct

As, we have proved above that

$f'(x) > 2.e^{2x} \ \forall \ x \in (0, \infty)$

$\Rightarrow$ option (D) is incorrect

$\therefore$ options (A) and (C) are correct

46. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

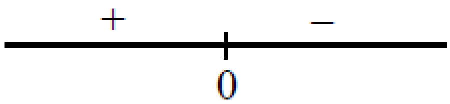
(A) f'(x) = 0 at exactly three points in (–π, π)
(B) f(x) attains its maximum at x = 0
(C) f(x) attains its minimum at x = 0
(D) f'(x) = 0 at more than three points in (–π, π)

Ans. (BD)

Expansion of determinant

$f(x) = \cos 2x + \cos 4x$

$f'(x) = -2\sin 2x - 4\sin 4x = -2\sin x(1 + 4\cos 2x)$



$\therefore$ maxima at x = 0

$f'(x) = 0 \Rightarrow x = \frac{n\pi}{2}, \cos 2x = -\frac{1}{4}$

$\Rightarrow$ more than two solutions

47. Let α and β be nonzero real numbers such that 2(cosβ – cosα) + cosα cosβ = 1. Then which of the following is/are true ?

(A) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$ (B) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

(C) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$ (D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

Ans. (AC)

$2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta-1=0$ .....(1)

use $\cos\beta=\dfrac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}$ and $\cos\alpha=\dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ in (1)

We get $\tan^2\left(\frac{\alpha}{2}\right)=3\tan^2\left(\frac{\beta}{2}\right)\Rightarrow\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\frac{\beta}{2}=0$ or $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

Hence (AC)

48. If $g(x)=\int_{\sin x}^{\sin(2x)}\sin^{-1}(t)\,dt$, then

(A) $g'\left(\frac{\pi}{2}\right)=-2\pi$ (B) $g'\left(-\frac{\pi}{2}\right)=2\pi$ (C) $g'\left(\frac{\pi}{2}\right)=2\pi$ (D) $g'\left(-\frac{\pi}{2}\right)=-2\pi$

Ans. BONUS

$$g(x)=\int_{\sin x}^{\sin 2x}\sin^{-1}t\,dt$$

$\Rightarrow g'(x)=2\sin^{-1}(\sin 2x)\times\cos 2x-\sin^{-1}(\sin x)\cos x$

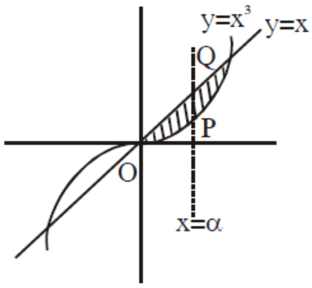
$\Rightarrow g'\left(\frac{\pi}{2}\right)=0$ & $g'\left(-\frac{\pi}{2}\right)=0$

No option matches the result

49. If the line x = α divides the area of region R = $\{(x, y)\in R^2 : x^3\le y\le x, 0\le x\le 1\}$ into two equal parts, then

(A) $\frac{1}{2}<\alpha<1$ (B) $\alpha^4+4\alpha^2-1=0$ (C) $0<\alpha\le\frac{1}{2}$ (D) $2\alpha^4-4\alpha^2+1=0$

Ans. (AD)



Area between $y = x^3$ and $y = x$

in $x \in (0, 1)$ is

$$A=\int_0^1\left(x-x^3\right)dx=\frac{1}{4}$$

Area of curve linear triangle $OPQ=\frac{A}{2}=\frac{1}{8}$

$$\Rightarrow\int_0^{\alpha}\left(x-x^3\right)dx=\frac{1}{8}\Rightarrow 2\alpha^4-4\alpha^2+1=0$$

$$\Rightarrow\left(\alpha^2-1\right)^2=\frac{1}{2}\Rightarrow\alpha^2=\frac{\sqrt{2}-1}{\sqrt{2}}$$

50. Let $f(x)=\frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x\neq 1$. Then

(A) $\lim_{x\to1^+} f(x)$ does not exist (B) $\lim_{x\to1^-} f(x)$ does not exist

(C) $\lim_{x\to1^-} f(x)=0$ (D) $\lim_{x\to1^+} f(x)=0$

Ans. (AC)

$$f(x)\begin{cases}(1-x)\cos\frac{1}{1-x}, & x<1\\ -(1+x)\cos\frac{1}{1-x}, & x>1\end{cases}$$

$\lim_{x\to1^+} f(x)=\text{d.n.e}, \lim_{x\to1^-} f(x)=0$

PARAGRAPH 1(51 & 52)

Let O be the origin, and $\overrightarrow{OX},\overrightarrow{OY},\overrightarrow{OZ}$ be three unit vectors in the direction of the sides $\overrightarrow{QR},\overrightarrow{RP},\overrightarrow{PQ}$, respectively, of a triangle PQR.

51. $|\overrightarrow{OX}\times\overrightarrow{OY}|=$

(A) sin(Q + R) (B) sin(P + R) (C) sin 2R (D) sin(P + Q)

Ans. (D)

$\overrightarrow{OX}=\frac{\overrightarrow{QR}}{QR}$

$\overrightarrow{OY}=\frac{\overrightarrow{RP}}{RP}$

$|\overrightarrow{Ox}=\overrightarrow{OY}|=\sin R=\sin(P+Q)$

52. If the triangle PQR varies, then the minimum value of cos(P + Q) + cos(Q + R) + cos(R + P) is

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $-\frac{5}{3}$

Ans. (B)

$-(\cos P+\cos Q+\cos R)\geq-\frac{3}{2}$ as we know cosP + cosQ + cosR will take its maximum value when

$P=Q=R=\frac{\pi}{3}$

PARAGRAPH 2(53 & 54)

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha\neq\beta$. For n = 0,1,2,...., let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b.

53. If $a_4 = 28$, then p + 2q =

(A) 14 (B) 7 (C) 12 (D) 21

Ans. (C)

$\alpha^2 = \alpha + 1 \Rightarrow \alpha^4 = 3\alpha + 2$

$\therefore a_4 = 28 \Rightarrow p\alpha^4 + q\beta^4 = p(3\alpha + 2) + q(3\beta + 2) = 28$

$\Rightarrow p(3\alpha + 2) + q(3 - 3\alpha + 2) = 28$

$\Rightarrow (3p - 3q) + 2p + 5q = 28$ (as $\alpha \in Q^c$)

$\Rightarrow p = q, 2p + 5q = 28 \Rightarrow p = q = 4$

$\therefore p + 2q = 12$

54. $a_{12} =$

(A) $2a_{11} + a_{10}$ (B) $a_{11} - a_{10}$ (C) $a_{11} + a_{10}$ (D) $a_{11} + 2a_{10}$

Ans. (C)

$\alpha^2 = \alpha + 1 \Rightarrow \alpha^n = \alpha^{n-1} + \alpha^{n-2}$

$\Rightarrow\ p\alpha^n + q\beta^n = p(\alpha^{n-1} + \alpha^{n-2}) + q(\beta^{n-1} + \beta^{n-2})$

$a_n = a_{n-1} + a_{n-2}$

$\Rightarrow\ a_{12} = a_{11} + a_{10}$