January 8 Shift 1 - Physics

1. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length l. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is

a. $\ell m \omega^2/(k-m \omega^2)$
b. $\ell m \omega^2/(k+m \omega^2)$
c. $\ell m \omega^2/(k-m \omega)$
d. $\ell m \omega^2/(k+m \omega)$

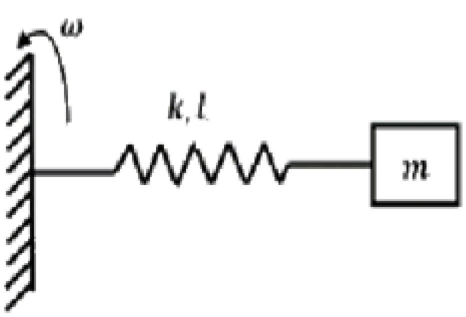
Solution:

The centripetal force is provided by the spring force.

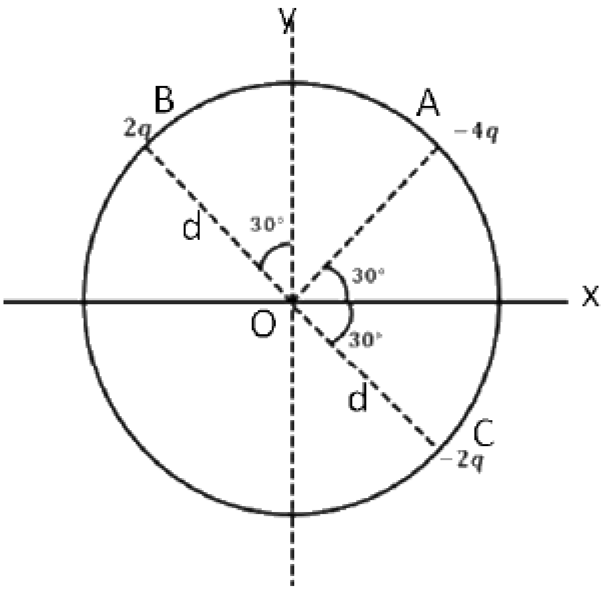
$m\omega^2(\ell + x) = kx$

$kx = m(\ell + x)\omega^2$

$x = m\ell\omega^2 /(k - m\omega^2)$



2. Three charged particles A, B and, C with charge -4q, +2q and -2q are present on the circumference of a circle of radius d. The charges particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x- direction is:

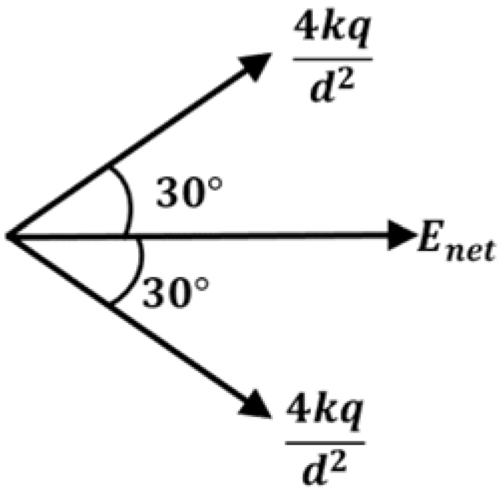


a. $\sqrt{3}q/4\pi\varepsilon_0 d^2$
b. $2\sqrt{3}q/\pi\varepsilon_0 d^2$
c. $\sqrt{3}q/\pi\varepsilon_0 d^2$
d. $3\sqrt{3}q/4\pi\varepsilon_0 d^2$

Solution:

Applying superposition principle,

$$\vec{E_{net}} = \vec{E_1} + \vec{E_2} + \vec{E_3}$$

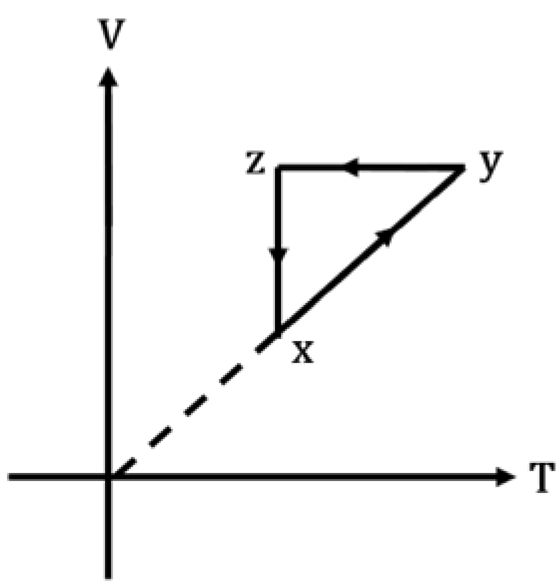


By symmetry, net electric field along the x-axis.

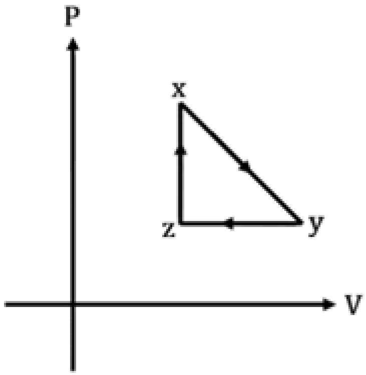
$$\left|\vec{E}_{net}\right| = \frac{4kq}{d^2} \times 2\cos 30^0 = \frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$$

---

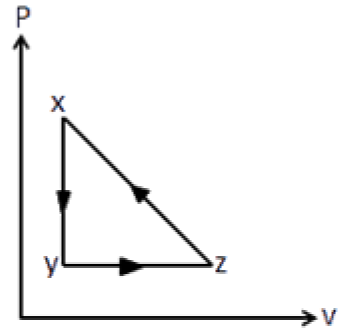
3. A thermodynamic cycle xyzx is shown on a V- T diagram .



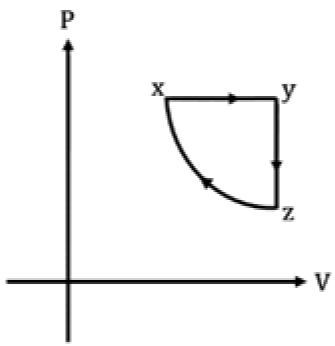
The P-V diagram that best describes this cycle is : (Diagrams are schematic and not upto scale)



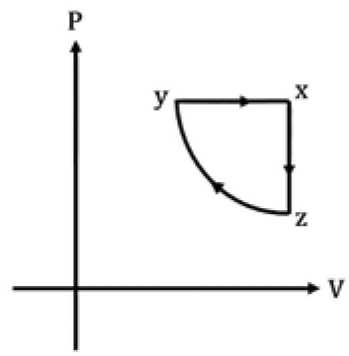
a.



b.



c.



d.

Solution:

For the given V - T graph

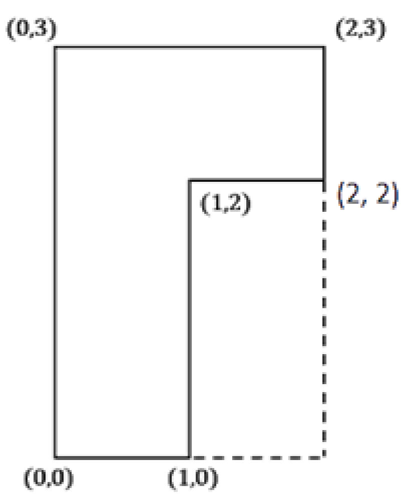
For the process x → y; V ∝ T; P =constant.

For the process y → z; V =constant

Only 'c' satisfies these two conditions.

---

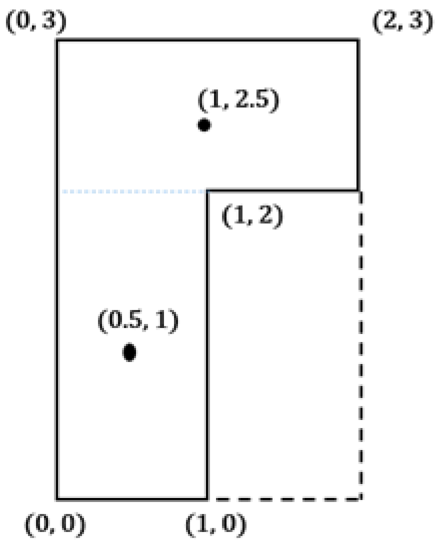
4. Find the co-ordinates of center of mass of the lamina shown in the figure below.



a. (0.75 m, 1.75 m)
b. (0.75 m, 0.75 m)
c. (1.25 m, 1.5 m)
d. (1 m, 1.75 m)

Solution:

The Lamina can be divided into two parts having equal mass m each.

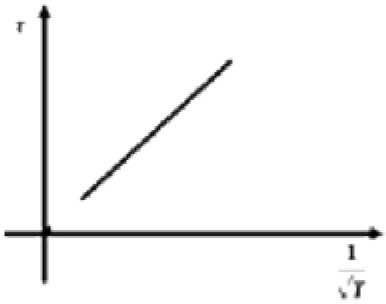


$$\vec{r}_{cm} = \frac{m\times\left(\frac{\hat{i}}{2}+\hat{j}\right)+m\times\left(\hat{i}+\frac{5\hat{j}}{2}\right)}{2m}$$

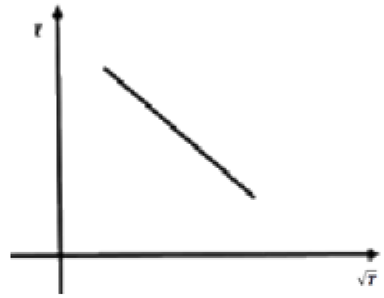
$$\vec{r}_{cm} = \frac{3}{4}\hat{i} + \frac{7}{4}\hat{j}$$

---

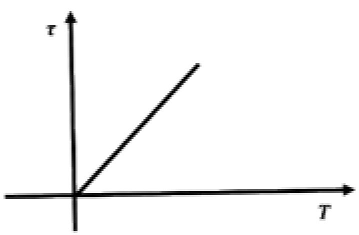
5. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graph are schematic and not drawn to scale) a.



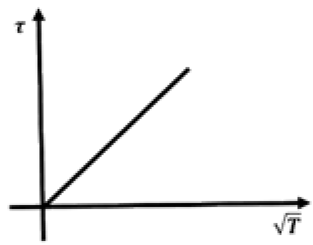
b.



c.



d.



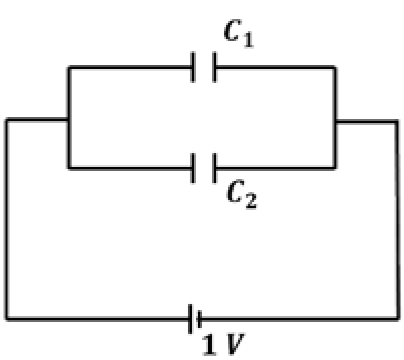
Solution:

$\tau \propto 1/\sqrt{T}$

6. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitor are individually connects to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times of that in C_1. If these capacitors are connected in series, their effective capacitance will be:

a. 1.6 μF
b. 3.2 μF
c. 4.2 μF
d. 8.4 μF

Solution:



Given that,

$C_1 + C_2 = 10$ μF ...(i)

$4(\frac{1}{2} C_1V^2) = \frac{1}{2} C_2V^2$

$4C_1 = C_2$ ...(ii)

From equations (i) and (ii)

$C_1 = 2\ \mu F$

$C_2 = 8\ \mu F$

If they are in series

$C_{eq} = C_1C_2/(C_1 + C_2)$

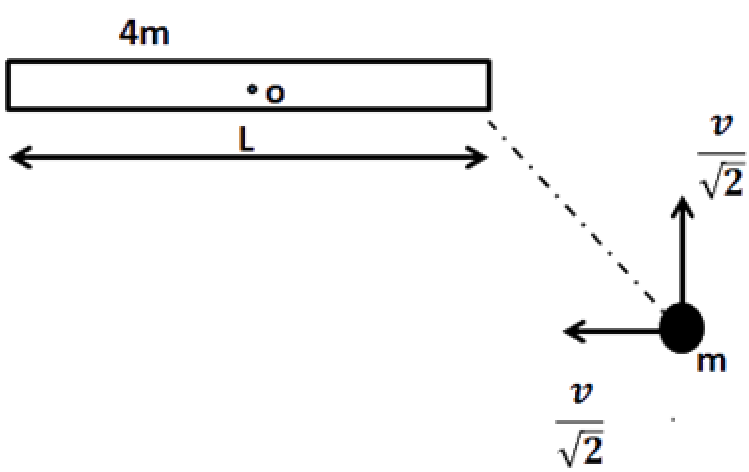
$= 1.6\ \mu F$

---

7. Consider a uniform rod of mass $4m$ and length L pivoted about its centre. A mass m is moving with a velocity v making angle θ=π/4 to the rod's long axis collides with one end of the rod and stick to it. The angular speed of the rod-mass system just after collision is

a. $3\sqrt{2}v/7L$
b. $4v/7L$
c. $3v/7\sqrt{2}L$
d. $3v/7L$

Solution:



There is no external torque on the system about the hinge point. So,

$$\vec{L}_1 = \vec{L}_f$$

$$\frac{mv}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4}\right] \times \omega$$

$= 6v/7\sqrt{2}L = 3\sqrt{2}\ v/7L$

---

8. When photons of energy 4 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is

a. 3 eV
b. 1.5 eV
c. 2 eV
d. 4 eV

Solution:

$$\lambda = \frac{h}{\sqrt{2(KE)m_B}}$$ = ∝ 1/√KE

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{KE_B}}{\sqrt{KE_A}}$$

$$\frac{1}{2} = \sqrt{\frac{T_A - 1.5}{T_A}}$$

T_A = 2eV

KE_B = 2 - 1.5 = 0.5 eV

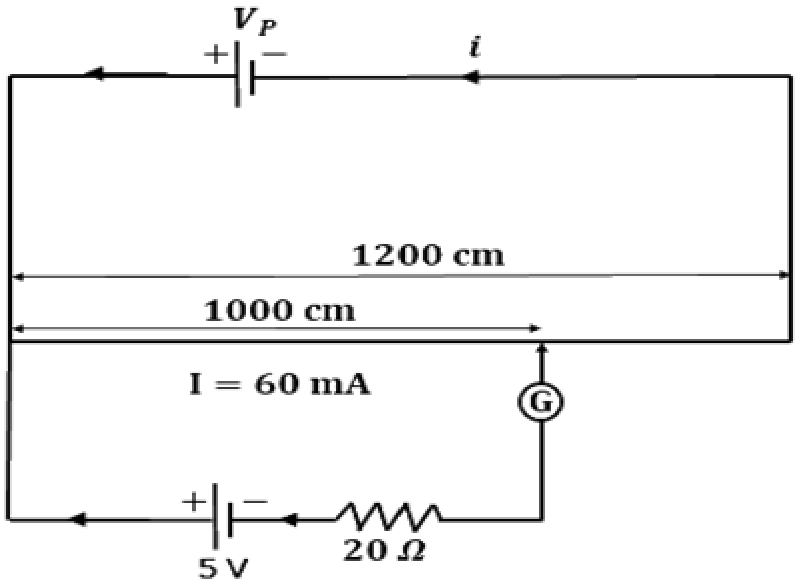
$\varnothing_B$ = 4.5 - 0.5 = 4 eV

---

9. The length of a potentiometer wire of length 1200 cm and it carries a current of 60 mA. For a cell of emf 5 V and internal resistance of 20Ω, the null point on it is found to be at 1000 cm. The resistance of whole wire is

a. 80Ω
b. 100Ω
c. 120Ω
d. 60Ω

Solution:



Let Resistance per unit length of potentiometer wire = λ

⇒ λ × 1000 × 60 × 10⁻³ = 5

⇒ λ 5 / 60

Resistance of potentiometer wire = 1200 × 5 / 60 = 100Ω

---

10. The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eyepiece?

a. 10 cm
b. 20 cm
c. 30 cm
d. 40 cm

Solution:

$m = f_0/f_e = 5$

$f_0 = 5f_e$

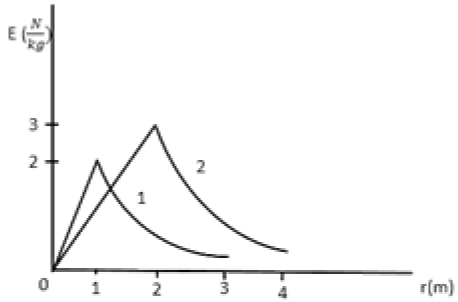
$f_0 + f_e = 5f_e + f_e = 6f_e$ = length of the tube

6fe = 60 cm

$f_e = 10$ cm

---

11. Consider two solid spheres of radii R_1= 1 m, R_2 = 2 m and masses M_1 & M_2, respectively. The gravitational field due to two spheres 1 and 2 are shown. The value of M_1/M_2 is



a. 1/6
b. 1/3
c. 1/2
d. 2/3

Solution:

Gravitation field will be maximum at the surface of a sphere. Therefore,

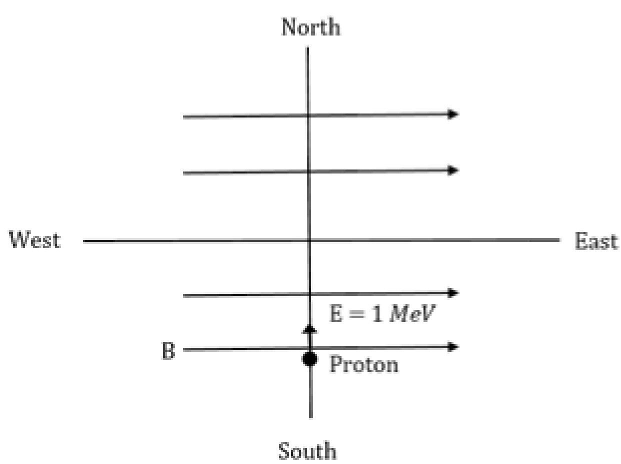
$GM_2/2^2 = 3$ and $GM_1/1^2 = 2$

$(M_2/M_1) \times ¼ = 3/2$

$M_1/M_2 = 1/6$

---

12. Proton with kinetic energy of 1 MeV moves from south to north. It gets an acceleration of 10^{12} m/s^2 by an applied magnetic field (west to east). The value of magnetic field: (Rest mass of proton is 1.6×10^{-27} kg)



a. 0.71 mT
b. 7.1 mT
c. 71 mT
d. 0.071 mT

Solution:

K. E = 1×10^6 eV = 1.6×10^{-13} J

= ½ $m_e v^2$

Where m_e is the mass of the electron = 1.6×10^{-27}

$1.6 \times 10^{-13} = ½ \times 1.6 \times 10^{-27} \times v^2$

$v = \sqrt{2} \times 10^7$ m/s

$Bqv = m_e a$

$B = (1.6 \times 10^{-27} \times 10^{12}) / (1.6 \times 10^{-19} \times \sqrt{2} \times 10^7)$

$= 0.71 \times 10^{-3}$ T

= 0.71 mT

---

13. If finding the electric field around a surface is given by $\left|\vec{E}\right| = \frac{q_{enclosed}}{\varepsilon_0 |A|}$ is applicable. In the formula ε_0 is permittivity of free space, A is area of Gaussian and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following equation?

a. Only when the Gaussian surface is an equipotential surface.

b. Only when $\left|\vec{E}\right|$ = constant on the surface.

c. Equipotential surface and $\left|\vec{E}\right|$ is constant on the surface

d. for any choice of Gaussian surfaces.

Solution:

The magnitude of the electric field is constant and the electric field must be along the area vector i.e. the surface is equipotential.

---

14. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant (h), speed of light (c), and gravitational constant (G) and Ampere (A) is

a. $h^{2/3} c^{5/3} G^{1/3} A^{-1}$
b. $h^2 c^{1/3} G^{3/2} A^{-1}$
c. $h^0 G^{-1} c^5 A^{-1}$
d. $h^{-2/3} c^{-1/3} G^{4/3} A^{-1}$

Solution:

$V = K(h)^a (I)^b (G)^c (c)^d$

Unit of stopping potential is (V_0) Volt.

We know $[h] = ML^2T^{-1}$

$[I] = A$

$[G] = M^{-1}L^3T^{-2}$

$[C] = LT^{-1}$

$[V] = ML^2T^{-3}A^{-1}$

$ML^2T^{-3}A^{-1} = (ML^2T^{-1})^a(A)^b\,(M^{-1}L^3T^{-2})^c(LT^{-1})^d$

$ML^2T^{-3}A^{-1} = M^{a-c}L^{2a+3c+d}T^{-a-2c-d}A^b$

$a - c = 1$

$2a + 3c + d = 2$

$-a - 2c - d = -3$ $b = -1$

On solving,

$c = -1$

$a = 0$

$d = 5$

$b = -1$

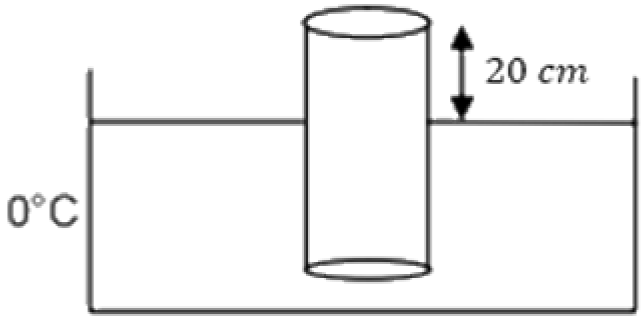
$V = K(h)^0(A)^{-1}(G)^{-1}(c)^5$

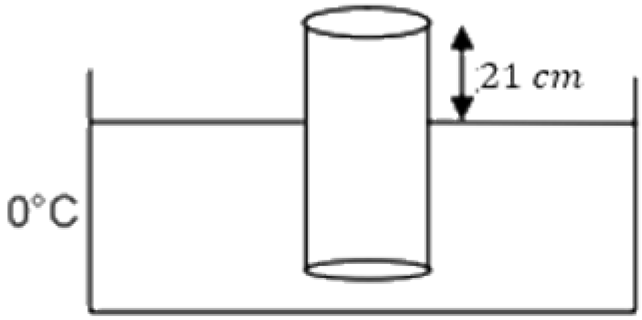
---

15. A leak proof cylinder of length 1 m, made of metal which has very low coefficient of expansion is floating in water at 0^o C such that its height above the water surface is 20 cm. When the temperature of water is increases to 4^o C, the height of the cylinder above the water surface becomes 21 cm. The density of water at $T = 4^o$ C relative to the density at $T = 0^o$ C is close to

a. 1.01
b. 1.03
c. 1.26
d. 1.04

Solution:





Since the cylinder is in equilibrium, it's weight is balanced by the Buoyant force.

mg = A(80)(ρ_{0oc})g

mg = A(79)(ρ_{4oc})g

$\rho_{4}{}^{o}c/\ \rho_{0oc}$ = 80/79 = 1.01

---

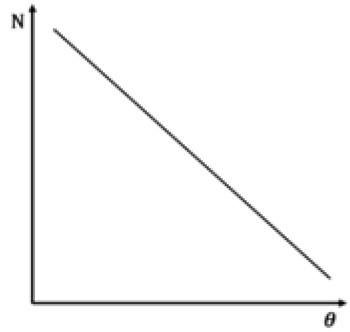
16. The graph which depicts the result of Rutherford gold foil experiment with α- particle is:

θ: Scattering angle

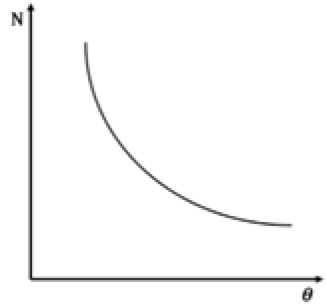
N : Number of scattered α – particles is detected

(Plots are schematic and not to scale)

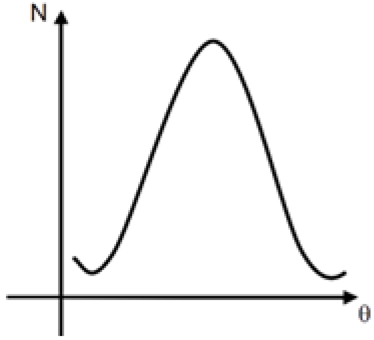
a.



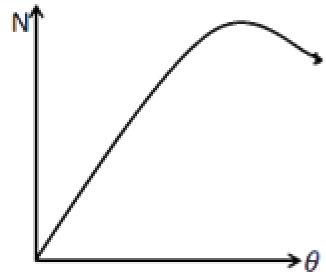
b.



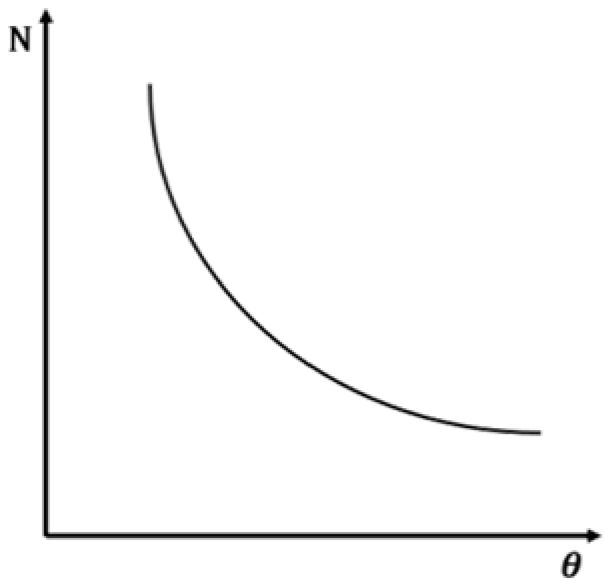
c.



d.

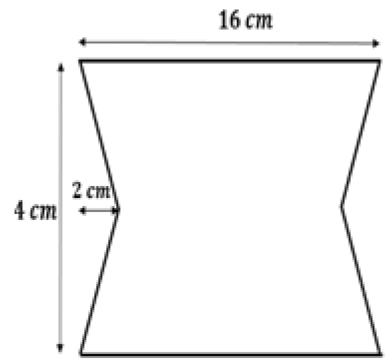


Solution:



$N \propto 1/\sin^4(\theta/2)$

17. At time t = 0 magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5 s, then induced EMF in the loop is:



a. 56 μV
b. 28 μV
c. 30 μV
d. 48 μV

Solution:

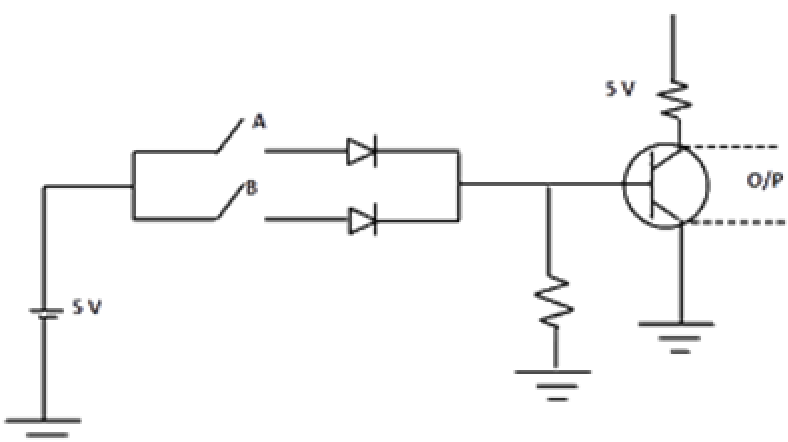
$$\epsilon = \left|\frac{-d\phi}{dt}\right| = \left|\frac{-A\,dB}{dt}\right|$$

$$(16 \times 4 - 4 \times 2)\frac{(1000-500)}{5} \times 10^{-4} \times 10^{-4}$$

= $56 \times 500 \times 10^{-8}/5$

= 56×10^{-6} V

18. Choose the correct Boolean expression for the given circuit diagram:



a. A.B
b. A + B

c. $\bar{A} + \bar{B}$
d. $\bar{A}.\bar{B}$

Solution:

First part of figure shown OR gate and second part of figure shown NOT gate.

So, Y = $\bar{A}.\bar{B}$

---

19. Consider a solid sphere of density $\rho(r) = \rho_0 (1- r^2/R^2)$, $0< r \le R$. The minimum density of a liquid in which it float is just

a. $(2/5)\ \rho_0$
b. $(2/3)\ \rho_0$
c. $\rho_0/5$
d. $\rho_0/3$

Solution:

Let the mass of the sphere be m and the density of the liquid be ρ_L

$\rho = \rho_0 (1 - r^2/R^2)$, $0< r \le R$

Since the sphere is floating in the liquid, buoyancy force (F_B) due to liquid will balance the weight of the sphere.

F = mg

$\rho_L (4\pi/3) R^3 g = \int\rho(4\pi r^2\, dr)g$

$\rho_L (4\pi/3) R^3 = \int \rho_0 (1 - r^2/R^2)\, 4\pi r^2 dr$

$\rho_L (4/3)\pi R^3 = \int_0^R \rho_0 4\pi(r^2 - \frac{r^4}{R^2})dr = \rho_0 4\pi \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R$

$\rho_L = (2/5)\ \rho_0$

---

20. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability 4/3 for this wavelength, will be

a. 15°
b. 30°
c. 45°
d. 60°

Solution:

If the speed of light in the given medium is V then,

$V = 1/\sqrt{(\mu \varepsilon)}$

We know that, $n = c/v$

$n = \sqrt{(\mu_r \varepsilon_r)} = 2$

$\sin \theta_c = ½$

$\theta_c = 30^0$

21. A body of mass m=0.10 kg has an initial velocity of $3\hat{i}\ m/s$. It collides elastically with another body, B of the mass which has an initial velocity of $5\hat{j}\ m/s$. After collision, A moves with a velocity $v = 4(\hat{i} + \hat{j})\ m/s$. The energy of B after collision is written as $(x/10)\ J$, the value of x is

Solution:

Mass of each object, $m_1 = m_2 = 0.1\ kg$

Initial velocity of 1st object, $u_1 = 5\ m/s$

Initial velocity of 2nd object, $u_2 = 3\ m/s$

Final velocity of 1st object, $V_1 = v = 4\hat{i} + 4\hat{j}\ m/s = \sqrt{(4^2+4^2)} = 16\sqrt{2}\ m/s$

For elastic collision, kinetic energy remains conserved

Initial kinetic energy (K_i) = final kinetic energy (K_f)

$½\ mu_1^2 + ½\ mu_2^2 = ½\ mV_1^2 + ½\ mV_2^2$

$½\ m(5)^2 + ½\ m(3)^2 = ½\ m(16\sqrt{2})^2 + ½\ mV_2^2$

$V_2 = \sqrt{2}$ m/s

Kinetic energy of second object = $½\ mV_2^2$

$= ½ \times 0.1 \times \sqrt{2}^2$

= 0.1

= 1/10 J

x = 1

22. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of lens material is 1.5, then the focal length of the lens (in cm) is

Solution:

Applying Lens makers' formula,

$1/f = (\mu - 1)[(1/R_1) - (1/R_2)]$

$R_1 = \infty$

$R_2 = -30$ cm

$\mu = 1.5$

$1/f = (1.5 - 1)[(1/\infty) - (1/-30)]$

$1/f = 0.5/30$

f = 60 cm

---

23. A particle is moving along the x-axis with its coordinate with time t given by $x(t) = -3t^2 + 8t + 10\ m$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y = 5 - 8t^3$ m. At t = 1 s, the speed of the second particle as measured in the frame of the first particle is given as √v . Then v (m/s) is

Solution:

$x = -3t^2 + 8t + 10$

$\vec{V_A} = (-6t + 8)\,\hat{i}$

$= 2\,\hat{i}$

$y = 5 - 8t^3$

$\vec{V_B} = -24t^2\,\hat{j}$

$$\left|\overrightarrow{V_{B/A}}\right| = \left|\overrightarrow{V_B} - \overrightarrow{V_A}\right| = \left|-24\hat{j} - 2\hat{i}\right|$$

$v = \sqrt{(24^2 + 2^2)}$

v = 580 m/s

---

24. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is __Hz.

Solution:

$V = \sqrt{(B/\rho)}$

$V_{pipe} / V_{air} = \sqrt{\frac{\frac{B}{2\rho}}{\frac{B}{\rho}}} = \frac{1}{\sqrt{2}}$

$V_{pipe} = V_{air}/\sqrt{2}$

$f_n = V_{pipe}(n+1)/2l$

$f_1 - f_0 = V_{pipe} / 2l$

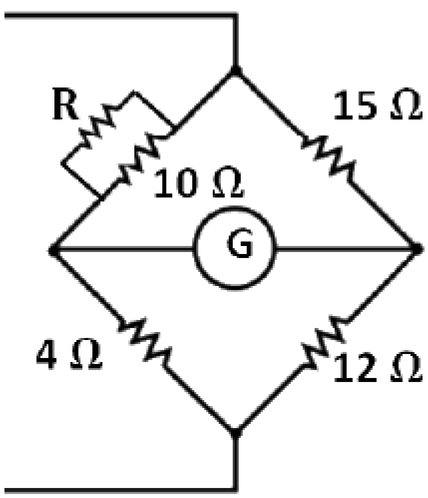
$= 300/2\sqrt{2}$

$= 106.06$ Hz (if $\sqrt{2} = 1.414$) ≈ 106 Hz

---

25. Four resistors of resistance 15 Ω, 12 Ω, 4Ω and 10Ω respectively in cyclic order to form a wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10 to balance the network is __.

Solution:



$[(10R)/(10+R)] \times 12 = 15 \times 4$

$R = 10\Omega$

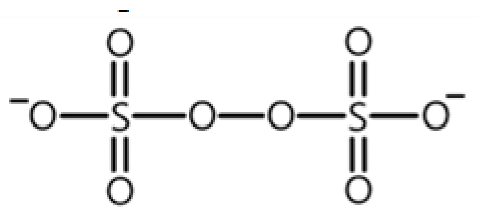
January 8 Shift 1 - Chemistry

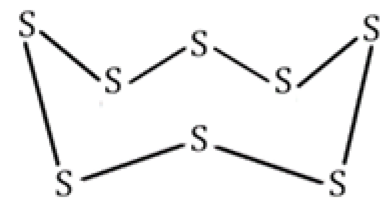
1. The number of bonds between sulphur and oxygen atoms in $S_2O_8^{2-}$ and number of bonds between sulphur and sulphur atoms in rhombic sulphur, respectively, are:

a. 8 and 6
b. 4 and 6
c. 8 and 8
d. 4 and 8

Solution:

Here, we have to count S – O single bonds as well as S = O in $S_2O_8^{2-}$, as each double bond also has one sigma bond. The structure of $S_2O_8^{2-}$ and S_8 is shown below:





---

2. The predominant intermolecular forces present in ethyl acetate, a liquid, are:

a. London dispersion, dipole-dipole and hydrogen bonding
b. hydrogen bonding and London dispersion
c. dipole-dipole and hydrogen bonding
d. London dispersion and dipole-dipole

Solution:

London dispersion forces (also called as induced dipole - induced dipole interactions), exist because of the generation of temporary polarity due to collision of particles and for this very reason, they are present in all molecules and inert gases as well.

Because of the presence of a permanent dipole, there will be dipole-dipole interactions present here.

There is no H that is directly attached to an oxygen atom, so H-bonding cannot be present.

3. For the Balmer series in the spectrum of H-atom,

$\overline{\nu} = R_H\,[\,(1/n_1^2) - (1/n_2^2)]$

The correct statements among (A) to (D) are:

A) The integer n_1= 2.

B) The ionization energy of hydrogen can be calculated from the wave number of these lines.

C) The lines of longest wavelength corresponds to n_2= 3.

D) As wavelength decreases, the lines of the series converge.

a. B, C, D
b. A, B, D
c. A, C, D
d. A, B, C

Solution:

A. is correct since the series studied in H-spectrum, including Balmer series, are de-excitation series or emission series. So, electrons get de-excited to n= 2 which means that n_{lower}= 2.
B. It is possible to obtain I.E. from the formula above, but since the question has stated the formula for the Balmer series, n_{lower} has been fixed as 2. So, it is not possible to calculate I.E. from it. To calculate I.E., we'll have to put n_{lower}= 1, which isn't possible here.
C. $\Delta E = hc/\lambda$
   With n_{lower} fixed as 2, ΔE increases as nhigher is increased. So, the last line of the Balmer series,i.e. from infinity to n= 2, will have the maximum energy in the series and thus, the lowest wavelength. Similarly, the first line in the series, i.e. from n= 3 to n= 2 will have the lowest energy in the series and thus, the highest wavelength. Which makes this statement correct.
D. As orbits with higher orbit number or those that are further away from the nucleus are considered, the energy gap in-between subsequent orbits decrease. Now, consider the following for example and with n_{lower} fixed as 2.

Energy of a photon released on transition from n= 100 to n= 2 will have similar energy to that of the photon that gets released on transition from n= 101 to n= 2, because energy of the 100th and the 101th orbit will be very close in value. That means they will also have very close values of wavelengths, which further implies that these two lines will be situated quite close to each other on the photographic plate.

In a similar fashion, we can see that as the nhigher increases, the lines start to converge together. And since, increasing the nhigher will indeed lead to an increase in the energy of the photon released, it will end up releasing photons of shorter wavelengths. Combining these two statements we can easily see that as the wavelength decreases, the spectral lines start to converge.

---

4. The first ionization energy (in kJ/mol) of Na, Mg, Al and Si, respectively,

a. 496, 737, 577, 786
b. 496, 577, 737, 786
c. 496, 577, 786, 737
d. 786, 737, 577, 496

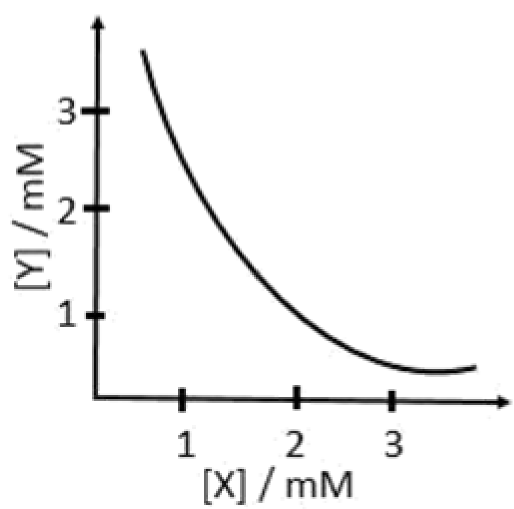
Solution:

The expected order is Na < Mg < Al < Si.

But the actual/experimental order turns out to be Na < Al < Mg < Si, because of the fully filled s-sub shell of magnesium and the s^2p^1 configuration of Al which makes it relatively easy for Al to lose its outermost electron.

---

5. The stoichiometry and solubility product of a salt with the solubility curve given below is, respectively:



a. X_2Y, $2 \times 10^{-9} M^3$
b. XY_2, $1 \times 10^{-9} M^3$
c. XY_2, $4 \times 10^{-9} M^3$
d. XY, $2 \times 10^{-6} M^3$

Solution:

(a) $X_2Y(s) \rightleftharpoons 2X^+ + Y^{2-}$

$Ksp = [X^+]^2[Y^{2-}] = 4 \times 10^{-6} \times 10^{-3} = 4 \times 10^{-9}$

(b) $XY_2(s) \rightleftharpoons X^{2+} + 2Y^-$

$Ksp = [X^{2+}][Y^-]^2 = 10^{-3} \times 4 \times 10^{-6} = 4 \times 10^{-9}$

(c) $XY_2(s) \rightleftharpoons X^{2+} + 2Y^-$

$Ksp = [X^{2+}][Y^-]^2 = 10^{-3} \times (2 \times 10^{-3})^2 = 4 \times 10^{-9}$

(d) $XY_{(s)} \rightleftharpoons X^+ + Y^-$

$Ksp = [X^+][Y^-] = 10^{-3} \times 10^{-3} = 10^{-6}$

---

6. The complex that can show fac- and mer-isomers is:

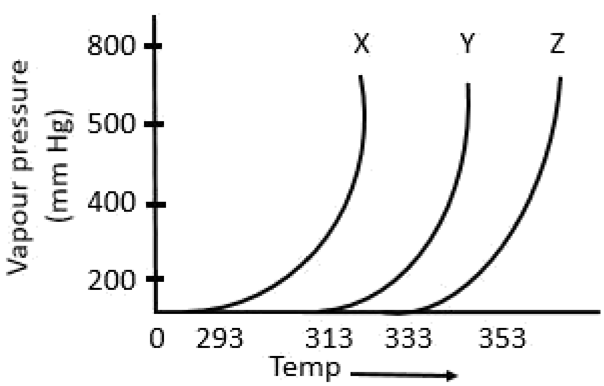
a. $[Co(NO_2)_3(NH_3)_3]$
b. $[PtCl_2(NH_3)_2$
c. $[Co(NH_3)_4Cl_2]$
d. $[CoCl_2(en)_2]$

Solution:

Facial and meridional geometrical isomerism is observed only in $[MA_3B_3]$ type complexes which is given in option a.

---

7. A graph of vapour pressure and temperature for three different liquids X, Y and Z is shown below:



The following inferences are made:

A) X has higher intermolecular interactions compared to Y

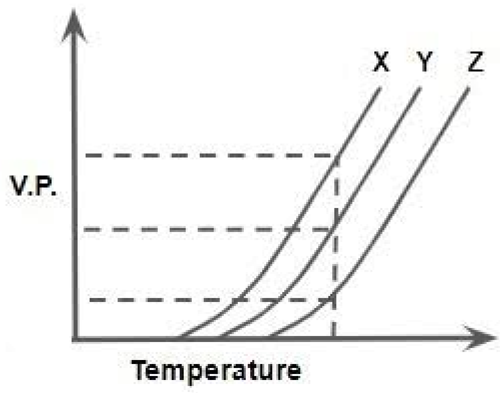
B) X has lower intermolecular interactions compared to Y

C) Z has lower intermolecular interactions compared to Y The correct inference(s) is/are:

a. C
b. A
c. B
d. A and C

Solution:

As shown in the plot below, for the same T, the vapour pressure of X is the highest and of Z is the lowest. Now, that means with the same average K.E. of X, Y and Z molecules, the X molecules are able to compensate their respective intermolecular forces better. So, X molecules have the highest vapour pressure. This implies that the intermolecular forces in X are the weakest among the three. The opposite could be said for Z as well.



---

8. As per Hardy-Schulze formulation, the flocculation values of the following for ferric hydroxide sol are in the order:

a. $AlCl_3 > K_3[Fe(CN)_6] > K_2CrO_4 > KBr = KNO_3$
b. $K_3\ [Fe(CN)_6\ ] < K_2CrO_4 < AlCl_3 < KBr < KNO_3$
c. $K_3\ [Fe(CN)_6\ ] < K_2CrO_4 < KBr = KNO_3 = AlCl_3$
d. $K_3\ [Fe(CN)_6\ ] > AlCl_3 > K_2CrO_4 > KBr > KNO_3$

Solution:

The minimum concentration of an electrolyte which is required to cause the coagulation or flocculation of a sol is known as flocculation value. Flocculation value is inversely proportional to coagulation power (coagulation power is directly proportional to the valency of the ions causing coagulation). $Fe(OH)_3$ sol is a positive sol and thus we have to consider the valency of the anions in the given electrolytes.

---

9. The rate of a certain biochemical reaction at physiological temperature (T) occurs 10^6 times faster with enzyme than without. The change in activation energy upon adding enzyme is:

a. − 6RT
b. − 6 × 2.303 RT
c. + 6RT
d. +6 × 2.303 RT

Solution:

$K_1 = Ae^{-E_{a_1}/RT}$ ....(1)

$K_2 = Ae^{-E_{a_2}/RT}$ ....(2)

Dividing equation 1 with equation 2, we get

$K_1/K_2 = e^{(Ea2-Ea1)/RT}$

$10^{-6} = e^{(Ea2-Ea1)/RT}$

Taking loge on both sides, we get

$\Delta E = E_{a2} - E_{a1} = -6 \times 2.303\ RT$

---

10. When gypsum is heated to 393K, it forms:

a. CaSO4. 1/2 H_2O
b. Dead burnt plaster
c. $CaSO_4$ $5H_2O$
d. Anhydrous $CaSO_4$

Solution:

$$CaSO_4.\ 2H_2O \xrightarrow{393K} CaSO_4.\frac{1}{2}H_2O$$

---

11. The third ionization enthalpy is minimum for:

a. Mn
b. Co
c. Ni
d. Fe

Solution:

Consider an element E

$E^{2+} \rightarrow E^{3+}$ would be the 3rd I.E. of the element E.

Electronic configuration of Mn is $[Ar]4s^23d^5$, Co is $[Ar]4s^23d^7$, Fe is $[Ar]4s^23d^6$, Ni is $[Ar]4s^23d^8$

Electronic configuration of Mn^{2+} is $[Ar]3d^5$, Co^{2+} is $[Ar]3d^7$, Fe^{2+} is $[Ar]3d^6$, Ni^{2+} is $[Ar]3d^8$

As it is evident from the above configurations of the E^{2+} for the given elements, Fe^{2+} would require the least amount of energy for removal of electron as it has the configuration $3d^6\ 4s^0$. That means that its E^{3+} form is the most stable among the four elements provided in their respective E^{3+} states, i.e., when compared, the next electron removal will require least amount of energy.

12. The strength of an aqueous NaOH solution is most accurately determined by titrating: (Note: consider that an appropriate indicator is used)

a. Aq. NaOH in a pipette and aqueous oxalic acid in a burette
b. Aq. NaOH in a volumetric flask and concentrated H_2SO_4 in a conical flask
c. Aq. NaOH in a burette and concentrated H_2SO_4 in a conical flask
d. Aq. NaOH in a burette and aqueous oxalic acid in a conical flask

Solution:

The standard solution is usually kept in burette. The oxalic acid is a primary standard solution while H_2SO_4 is a secondary standard solution.

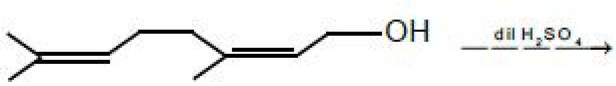
13. The decreasing order of reactivity towards dehydrohalogenation (E_1) reaction of the following compounds is:

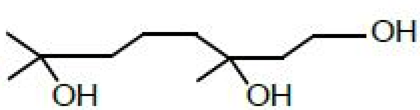
a. B > A > D > C
b. B > D > C > A
c. B > D > A > C
d. D > B > C > A

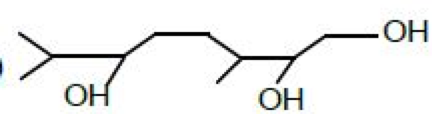
Solution:

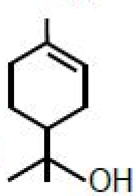
In E_1mechanism, the rate determining step is formation of carbocation. So, stability of carbocation formed decides the rate. In option D, the cation formed is resonance stabilised. In option C, the cation formed is a 2 o carbocation. In option A and B, the carbocations formed are 1 o but there is a chance of rearrangement in option b and after the rearrangement, the carbocation formed in option b will be allylic. So, the order of reaction is as follows: D > B > C > A.

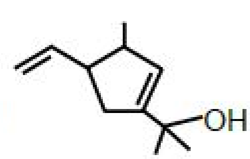
14. Major product in the following reaction is:



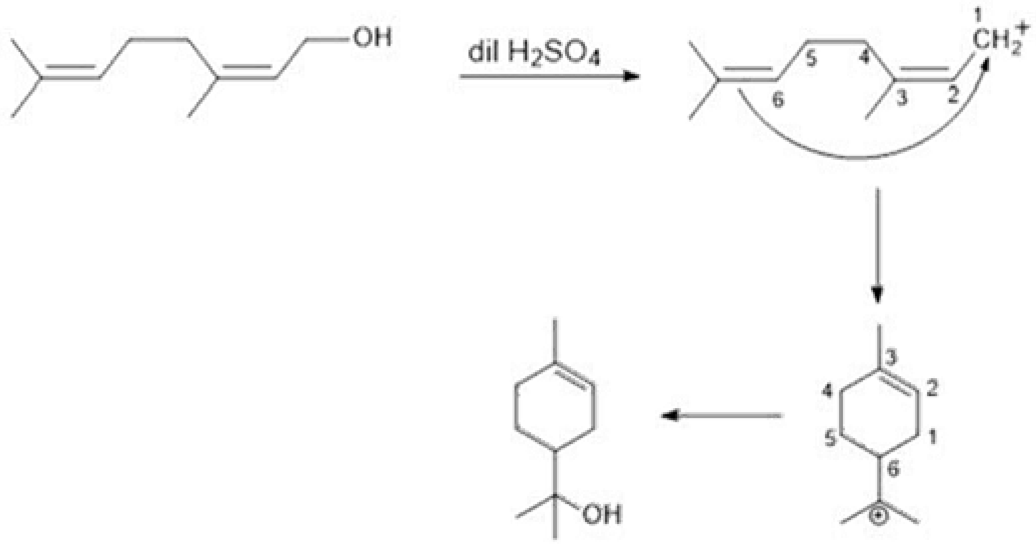
(1) 

(2) 

(3) 

(4) 

Solution:



15. **15.** Arrange the following compounds in increasing order of C—OH bond length: methanol, phenol, p-ethoxyphenol

a. Phenol < methanol < p-ethoxyphenol
b. methanol < p-ethoxyphenol < phenol
c. Phenol < p-ethoxyphenol < methanol
d. methanol < phenol < p-ethoxypheno

Solution:

In methanol, there is no resonance. In phenol, there is resonance. In p-Ethoxyphenol, there is resonance involved but the involvement of lone pair of oxygen in OH group is poor as compared with phenol due to the presence of lone pair oxygen in OCH_3 group which are also involved in resonance. So, partial double bond character develops in C—OH bond of phenol and p-ethoxyphenol but in case of p-ethoxyphenol, resonance is poor as compared to phenol. So, bond length follows the order: methanol > p-ethoxyphenol > phenol

16. Among the gases (i) – (v), the gases that cause greenhouse effect are:

i. CO_2

ii. H_2O

iii. CFC

iv. O_2

v. O_3

a. i, ii iii and iv
b. i, iii, iv and v
c. i and iv

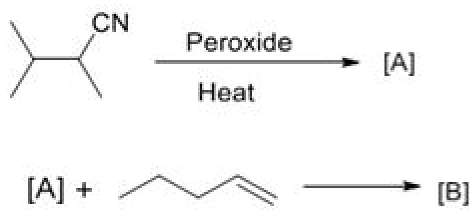
d. i, ii, iii and v

Solution:

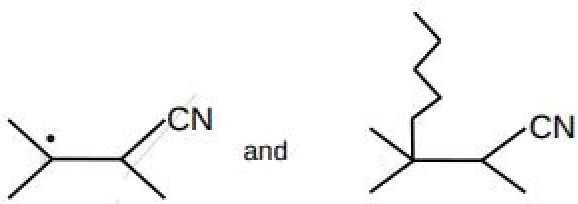
CO_2, O_3, H_2O vapours and $CFC's$ are green house gases.

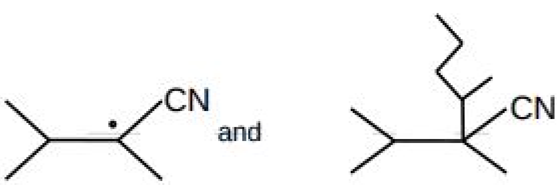
---

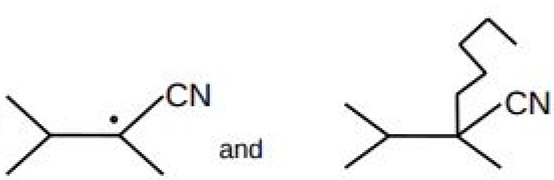
17. The major products A and B in the following reactions are:



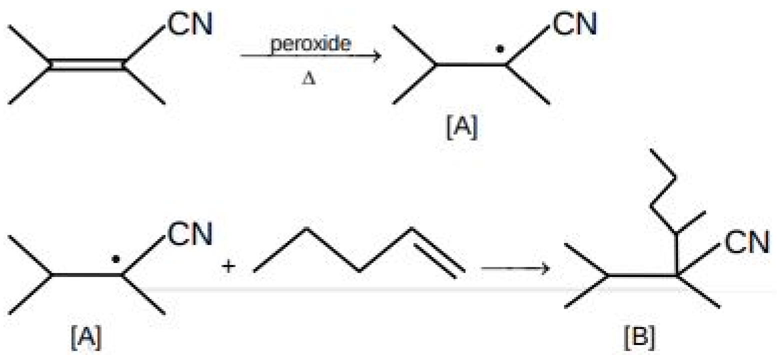
(a) 

(b) 

(c) 

(d) 

Solution:



[A] is more stable radical and undergoes Markovnikov addition to form [B].

---

18. A flask contains a mixture of isohexane and 3-methylpentane. One of the liquids boils at 63 °C while the other boils at 60 °C. What is the best way to separate the two liquids and which one will be distilled out first?

a. Fractional distillation, isohexane
b. Simple distillation, 3-methylpentane
c. Fractional distillation, 3-methylpentane
d. Simple distillation, isohexane

Solution:

When the difference between the B.P. of the two liquids is less than around 40 ^oC, fractional distillation is more efficient. The difference between the boiling points of isohexane and 3-methylpentane is only 3 degrees. So, fractional distillation is the best suitable method. Since, isohexane has a lower boiling point, it comes out first.

---

19. Which of the given statement is not true for glucose?

a. The pentacetate glucose does not react with hydroxylamine to give oxime
b. Glucose reacts with hydroxylamine to form oxime.
c. Glucose gives Schiff's test for aldehyde.
d. Glucose exists in two crystalline forms alpha and beta.

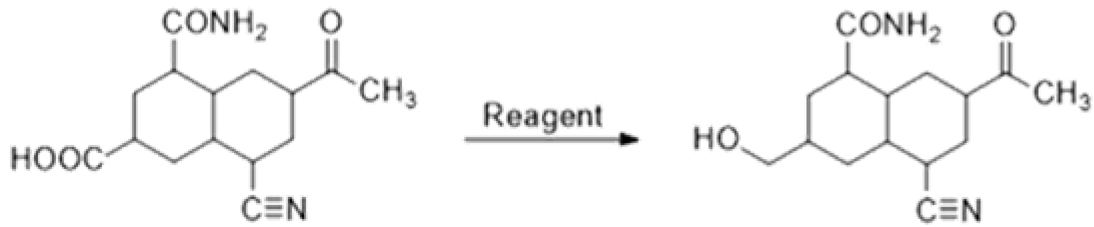
Solution:

Glucose exists in two crystalline forms alpha and beta which are anomers of each other.

Glucose does not react with Schiff’s reagent because after the internal cyclisation, it forms either alpha- anomer or beta-anomer. In these forms, free aldehydic group is not present.

Glucose forms open chain structure in aqueous solution which contains aldehyde at chain end. This aldehydic group reacts with NH_4OH to form oxime. On the other hand, glucose penta acetate being a cyclic structure even in aqueous form does not have terminal carbonyl group. Therefore it will not react with NH_4OH.

20. The reagent used for the given conversion is:



a. B_2H_6
b. $LiAlH_4$
c. $NaBH_4$
d. H_2, Pd

Solution:

B_2H_6 does not reduce amide, carbonyl group and cyanide. It selectively reduces carboxylic acid to alcohol. So, for this conversion, it is the best suitable reagent.

---

21. The volume (in mL) of 0.125 M $AgNO_3$ required to quantitatively precipitate chloride ions in 0.3 g of $[C(NH_3)_6]Cl_3$ is ____.

$[Co(NH_3)_6]Cl_3$ = 267.46 g/mol

$MAgNO_3$ = 169.87 g/mpl

Solution:

To react completely with one mole of $[ML_6]l_3$, 3 moles of $AgNO_3$ is required.

0.3 g $[ML_6]l_3$ means (0.3/267.46) moles of $[ML_6]Cl_3$.

So, moles of $AgNO$3 required will be (0.3×3)/267.46 moles

To find the volume, (0.3×3)/267.46 = 0.125× (L)

(L) = 0.02692

(mL) = 26.92

---

22. What will be the electrode potential for the given half cell reaction at pH= 5?

$2H_2O \rightarrow O_2 + 4H^+ + 4e^-; E° = -1.23\ V$

(R=8.314 J $mol^{-1}K^{-1}$; temp.=298 K; oxygen under std. atm. Pressure of 1 bar.)

Solution:

$2H_2O \rightarrow O_2 + 4H^+ + 4e^-; E° = -1.23\ V$

$$E_{cell} = E^0_{cell} - \frac{RT}{nF}\ln Q$$

at 1 bar & 298 K

$$\frac{2.303RT}{F} = 0.059$$

pH = 5 $\Rightarrow [H^+] = 10^{-5}$ M

$E^\circ_{oxidation} = -1.23$ volt

$$E_{cell} = -1.23 - \frac{0.059}{4}\log[H^+]^4$$

$$E_{cell} = -1.23 - \frac{0.059}{4}\log(10^{-5})^4$$

$= -1.23 + 0.059 \times 5 = -0.935$ V

---

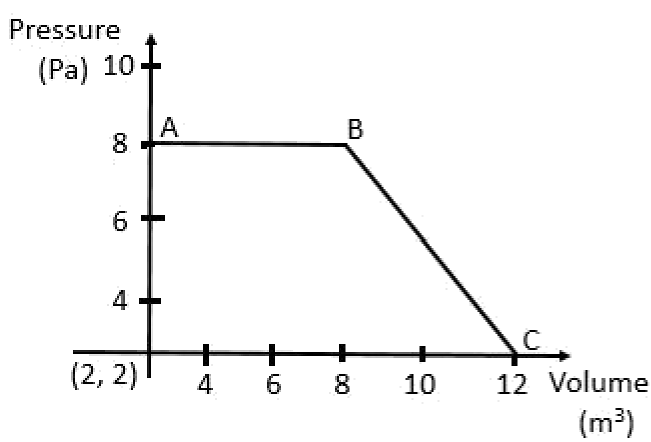
23. Ferrous sulphate heptahydrate is used to fortify foods with iron. The amount (in grams) of the salt required to achieve 10 ppm of iron in 100 kg of wheat is ______. Atomic weight: Fe=55.85; S=32.00; O=16.00)

Solution:

10 ppm of Fe means 10 g of Fe in 10^6 g of wheat. So, for 100 kg i.e.,10^5 g of wheat. Fe needed is 1 g. So, for 1 g of Fe, the mass of $FeSO_4 . 7H_2O$ required is 278/56 = 4.96 g.

---

24. The magnitude of work done by gas that undergoes a reversible expansion along the path ABC shown in figure is



Solution:

Work done by the gas

= The area under the curve

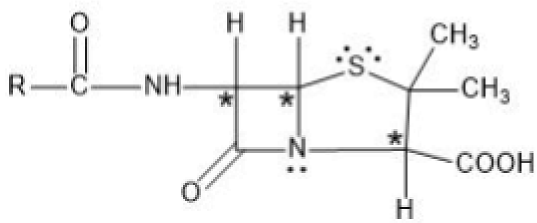
= (Area of the square) + (Area of the triangle)

= 48 J

25. The number of chiral centres in Penicillin is ____.

Solution:

The structure of penicillin is shown below:



So, the number of chiral centers= 3

January 8 Shift 1 - Maths

1. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent?

a. (4, 6)
b. (3, 4)
c. (1, 0)
d. (4, 3)

Solution:

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$$

$R_3 \rightarrow R_3 - 2R_1 + 2R_2$

D = $\begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix}$ =0

For inconsistent system, one of D_x, D_y, D_z should not be equal to 0

Dx = $\begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix}$

Dy = $\begin{vmatrix} 3 & \mu & 5 \\ 1 & 2 & 3 \\ 4 & \delta & 4 \end{vmatrix}$

Dz = $\begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix}$

For inconsistent system, $2\mu \neq \delta + 2$

∴ The system will be inconsistent for μ = 4, δ = 3.

---

2. Let y = (x) be a solution of the differential equation,

$\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$ .If y (1/2) = √3/2, then y(-1/√2) is equal to

a. -1/√2
b. -√3/2
c. 1/√2
d. √3/2

Solution:

$$\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0,$$

$$\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$$

$\sin^{-1}y + \sin^{-1}x = c$

If x = ½, y = √3/2 then,

$\sin^{-1}(\sqrt{3}/2) + \sin^{-1}(1/2) = c$

$(\pi/3) + (\pi/6) = c$

Therefore, $c = \pi/2$

$\sin^{-1}y = \pi/2 - \sin^{-1}x = \cos^{-1}x$

$\sin^{-1}y = \cos^{-1}(1/\sqrt{2})$

$\sin^{-1}y = \pi/4$

$y(1/\sqrt{2})) = 1/\sqrt{2})$

---

3. If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$, ${}^{21}C_r$ respectively, then:

a. (a/11) = (b/22) = (c/42)
b. (a/10) = (b/11) = (c/42)
c. (a/11) = (b/22) = (c/21)
d. (a/10) = (b/11) = (c/21)

Solution:

We know that, ${}^nC_r$ is maximum when $r = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n+1}{2} \text{ or } \frac{n-1}{2}, & n \text{ is odd} \end{cases}$

Therefore, max $({}^{19}C_p) = ({}^{19}C_9) = a$

max $({}^{20}C_p) = {}^{20}C_{10} = b$

max $({}^{20}C_r) = {}^{21}C_{11} = c$

$$\therefore \frac{a}{{}^{19}C_9} = \frac{b}{\frac{20}{10} \times {}^{19}C_9} = \frac{c}{\frac{21}{11} \times \frac{20}{10} \times {}^{19}C_9}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{2} = \frac{c}{\frac{42}{11}}$$

$$\Rightarrow \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

---

4. Which of the following is a tautology?

a. $(P \land (P \longrightarrow Q)) \longrightarrow Q$
b. $P \land (P \lor Q)$
c. $(Q \rightarrow (\land (P \rightarrow Q))$
d. $P \lor (P \land Q)$

Solution:

$(P \land (P \longrightarrow Q)) \longrightarrow Q$

$= (P \wedge (\sim P \vee Q)) \longrightarrow Q$

$= [(P \wedge \sim P) \vee (P \wedge Q)] \longrightarrow Q$

$= P \wedge Q \longrightarrow Q$

$= \sim (P \wedge Q) \vee Q$

$= \sim P \vee \sim Q \vee Q$

$= T$

---

5. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be such that for all $x \in \mathbf{R}$, $(2^{1+x} + 2^{1-x})$, f(x) and $(3^x + 3^{-x})$ are in A.P., then the minimum value of f(x) is:

a. 0
b. 4
c. 3
d. 2

Solution:

$2^{1-x} + 2^{1+x}$, (x), $3^x + 3^{-x}$ are in A.P.

$$f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{3^x + 3^{-x}}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Also, Applying A.M. ≥ G.M. inequality, we get

$$\frac{3^x + 3^{-x}}{2} \geq \sqrt{3^x 3^{-x}}$$

$$\frac{3^x + 3^{-x}}{2} \geq 1 \text{ -------- (1)}$$

Applying A.M. ≥ G.M. inequality, we get

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} \times 2^{1-x}}$$

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq 2 \text{ -------- (2)}$$

Adding (1) and (2), we get

f(x) ≥ 1 + 2 = 3

Thus, minimum value of f(x) is 3.

---

6. The locus of a point which divides the line segment joining the point (0, −1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1: 2, is:

a. $9x^2 - 12y = 8$
b. $4x^2 - 3y = 2$
c. $x^2 - 3y = 2$
d. $9x^2 - 3y = 2$

Solution:

Let point P be $(2t, t^2)$ and Q be (h, k).

$$h = \frac{2t}{3}, k = \frac{-2+t^2}{3}$$

Now, eliminating t from the above equations we get:

$3k+2 = (3h/2)^2$

Replacing h and k by x and y, we get the locus of the curve as $9x^2 - 12y = 8$.

---

7. For $a > 0$, let the curves $C_1: y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b$ $(0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line x = b bisects the area bounded by the curves, C_1 and C_2 , and the area of ΔOQR=1/2,then'a' satisfies the equation

a. $x^6 - 12x^3 + 4 = 0$
b. $x^6 - 12x^3 - 4 = 0$
c. $x^6 + 6x^3 - 4 = 0$
d. $x^6 - 6x^3 + 4 = 0$

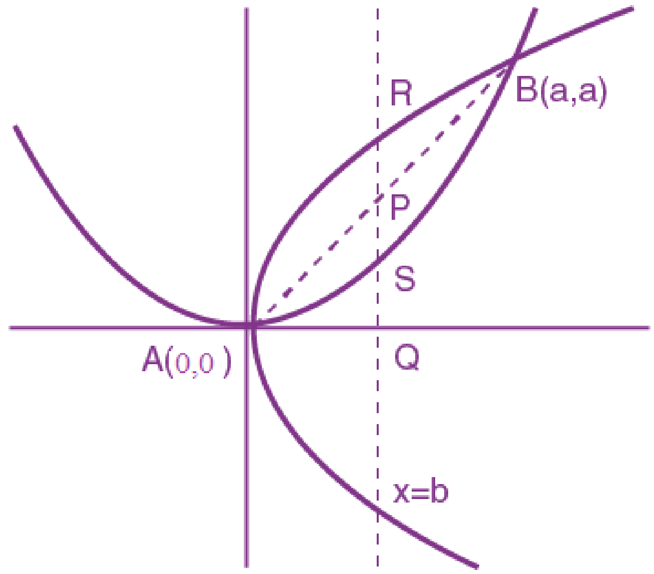
Solution:

Given, (ΔAPQ)=1/2

$\Rightarrow$ $1/2 \times b \times b = 1/2$

$\Rightarrow b=1$

[redacted]



As per the question

$$\int_0^1 \left(\sqrt{ax} - \frac{x^2}{a}\right) dx = \frac{1}{2} \int_0^a \left(\sqrt{ax} - \frac{x^2}{a}\right) dx$$

(2/3)√a – (1/3a) = a^2/6

$2a\sqrt{a}-1=a^3/2$

$\Rightarrow 4a\sqrt{a}=2+a^3$

$\Rightarrow 16a^3=4+a^6+4a^3$

$\Rightarrow a^6-12a^3+4=0$

8.The inversion function of f(x) = $\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}, x\epsilon(-1,1), is$

a. $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

b. $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

c. $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

d. $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

Solution:

$$f(x) = \frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}} = \frac{8^{4x-1}}{8^{4x+1}}$$

Put, y = $\frac{8^{4x-1}}{8^{4x+1}}$

Applying componendo-dividendo on both sides

$\frac{y+1}{y-1} = \frac{2\times 8^{4x}}{-2}$

$8^{4x} = (1+y)/(1-y)$

$\Rightarrow x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$

$f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right) = \frac{1}{4}\log_8 e\left(\log_e\frac{1+x}{1-x}\right)$

---

9. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ **is equal to**

a. e
b. $1/e^2$
c. 1/e
d. e^2

Solution:

Let,L= $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ (intermediate from 1^∞)

$$L = e^{\lim_{x\to 0}\frac{1}{x^2}\left(\frac{3x^2+2}{7x^2+2}-1\right)}$$

$$L = e^{\lim_{x\to 0}\frac{1}{x^2}\left(\frac{-4x^2}{7x^2+2}\right)}$$

$= 1/e^2$

---

10. Let f(x) = (sin($\tan^{-1} x$) + sin($\cot^{-1} x$))^2 – 1, where $|x|$ > 1.

If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(sin^{-1}f(x))$ **and y(√3) = π/6, then y(-(√3) is equal to**:

a. π/3
b. 2π/3
c. - π/6
d. 5 π/6

Solution:

f(x) = [sin($\tan^{-1} x$) + sin($\cot^{-1} x$)]^2 -1

Put $\tan^{-1} x = \phi$, where $\phi \in (-\pi/2, -\pi/4) \cup (\pi/4, \pi/2)$

= [sin($\tan^{-1} x$) + sin($\cot^{-1} x$) ^{2}-1 = [sin ϕ+cos ϕ]^2-1

= 1+2sin ϕcos ϕ -1 = sin2ϕ = (2x/1+x^2)

It is given that $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(sin^{-1}f(x)))$

$$\frac{dy}{dx} = -\frac{1}{1+x^2}, for\ |x| > 1$$

⇒ x>1 and x<-1

To get the value of (−√3), we have to integrate the value of dy/dx. To integrate the expression, the

interval should be continuous. Therefore, we have to integrate the expression in both the intervals.

⇒ $y = -\tan^{-1} x + C_1$, for $x > 1$ and $y = -\tan^{-1} x + C_2$, for $x < -1$

For $x > 1$,C_1 =π/2

Since y(√3) = π/6 is given.

But C_2 can't be determined as no other information is given for $x < -1$. Therefore, all the options can be true as C_2 can't be determined.

---

11. If the equation, $x^2 + bx + 45 = 0$ ($b \in \boldsymbol{R}$) has conjugate complex roots and they satisfy $|z + 1| = 2\sqrt{10}$, then :

a. $b^2 + b = 12$
b. $b^2 - b = 42$
c. $b^2 - b = 30$
d. $^2 + b = 72$

Solution:

Given $x^2 + bx + 45 = 0$, $b \in \boldsymbol{R}$, let roots of the equation be $p \pm iq$

Then, sum of roots = $2p = -b$

Product of roots = $p^2 + q^2 = 45$

As $p \pm iq$ lies on $|z + 1| = 2\sqrt{10}$, we get

$(p + 1)^2 + q^2 = 40$

⇒ $p^2 + q^2 + 2p + 1 = 40$

⇒ $45 - b + 1 = 40$

⇒ $b = 6$

⇒ $b^2 - b = 30$.

---

12. The mean and standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and standard deviation become half of their original values, then q is equal to:

a. −20
b.−5
c. 10
d.−10

Solution:

If mean $\bar{x}$ is multiplied by p and then q is subtracted from it, then new mean $\bar{x}' = p\bar{x} - q$

$$\bar{x}' = \frac{1}{2}\bar{x}$$

And x= 10

⇒ 10 = 20 p −q--------(1)

If standard deviation is multiplied by p, new standard deviation (σ') is $|p|$ times of the initial standard deviation (σ).

$$\sigma' = |p|\,\sigma$$

$$\Rightarrow \frac{1}{2}\sigma = |p|\,\sigma$$

$$|p| = 1/2$$

If p =1/2 ,q = 0

If p = -1/2 , q = -20

13.

If $\int \frac{\cos x}{\sin^3 x(1+\sin^6 x)^{\frac{2}{3}}} dx = f(x)(1+\sin^6 x)^{\frac{1}{\lambda}} + c$, where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to :

a. $-\frac{9}{8}$

b. $\frac{9}{8}$

c. 2

d. -2

Solution:

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\therefore \int \frac{dt}{t^3(1+t^6)^{\frac{2}{3}}} = \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Let $1+\frac{1}{t^6} = u \Rightarrow -6t^{-7}dt = du$

$$\Rightarrow \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}} = -\frac{1}{6}\int \frac{du}{u^{\frac{2}{3}}} = -\frac{3}{6}u^{\frac{1}{3}} + c = -\frac{1}{2}\left(1+\frac{1}{t^6}\right)^{\frac{1}{3}} + c$$

$$= -\frac{(1+\sin^6 x)^{\frac{1}{3}}}{2\sin^2 x} + c = f(x)(1+\sin^6 x)^{\frac{1}{\lambda}}$$

$$\therefore \lambda = 3 \text{ and } f(x) = -\frac{1}{2\sin^2 x}$$

$$\Rightarrow \lambda f\left(\frac{\pi}{3}\right) = -2.$$

---

14. Let A and B be two independent events such that P(A) =1/3 and P(B) = 1/6. Then, which of the following is TRUE ?

a. P($A/(A \cup B)$) = 1/4
b. P(A/B') = 1/3
c. P(A/B) = 2/3
d. P(A'/B') = 1/3

Solution:

If X and Y are independent events, then

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)}$$

$$= \frac{P(X)P(Y)}{P(Y)} = P(X)$$

Therefore,P(A/B) = P(A) = 1/3

⇒ P(A/B') = P(A) = 1/3

15. If volume of parallelepiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$,

$\hat{v} == \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the

edges $\vec{u}$ and $\vec{w}$ then, cos θ can be:

a. 7/(6√6)
b. 5/7
c. 7/(6√3)
d. 5/(3√3)

Solution:

Volume of parallelepiped = $\begin{bmatrix}\vec{u} & \vec{v} & \vec{w}\end{bmatrix}$

⇒ $\begin{vmatrix}1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1\end{vmatrix} = \pm 1$

⇒ λ = 2 or 4

For λ = 4,

$$cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$$

16. Let two points be A(1, −1) and B(0, 2). If a point P(x', y') be such that the area of ΔPAB = 5 sq.

units and it lies on the line, 3x + y − 4λ = 0, then the value of λ is :

a. 4
b. 1

c. −3
d. 3

Solution:

Area of triangle is

$$A = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ x' & y' & 1 \end{vmatrix} = \pm 5$$

$$\Rightarrow \Rightarrow (2 - y') - x' - 2x' = \pm 10$$

$$\Rightarrow -3x' - y' + 2 = \pm 10$$

3x’+y’=12 or 3x’+y’=-8

⇒ λ = 3 or −2

---

17. The shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

And $\frac{x+3}{3} = \frac{y+7}{2} = \frac{z-6}{1}$ **is**

a. 2√30
b. (7/2)√30
c. 3
d. 3√30

Solution:

$$\vec{AB} = -3\hat{i} - 7\hat{j} + 6\hat{k} - (3\hat{i} + 8\hat{j} + 3\hat{k}) = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\vec{p} = -3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{q} = -3\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = -6\hat{i} - 15\hat{j} + 9\hat{k}$$

Shortest distance = $\frac{\left|\vec{AB}(\vec{p}\times\vec{q})\right|}{|\vec{p}\times\vec{q}|}$

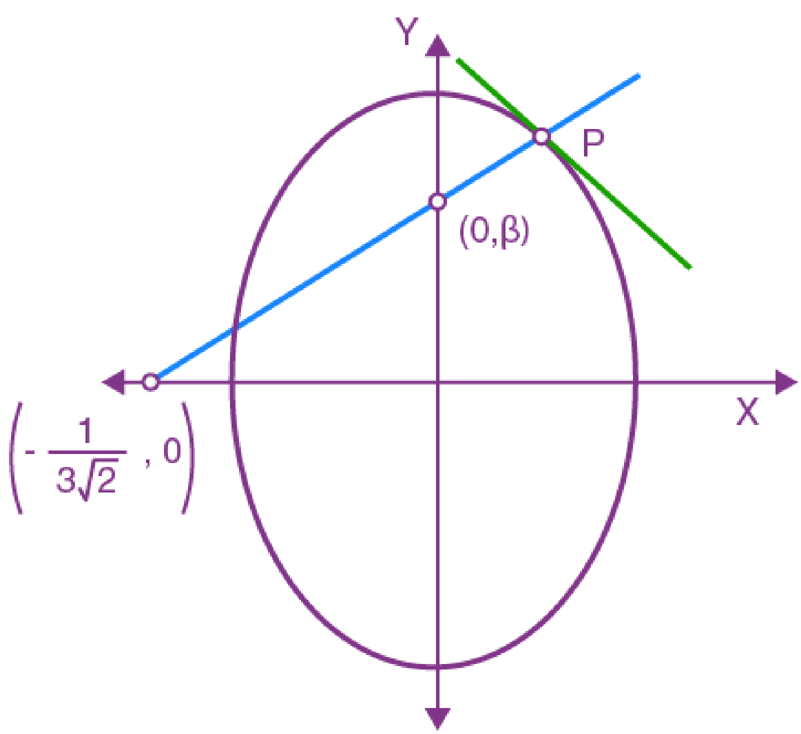
= $\frac{|36+225+9|}{\sqrt{36+225+9}} = 3\sqrt{30}$

---

18. Let the line y = mx and the ellipse $2x^2$ + y^2 = 1 intersect a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at (−1/3√2,0) and (0, β), then β is equal to

a. 2/√3
b. 2/3
c. 2√2/3
d. √2/3

Solution:



Let $P \equiv (x_1, y_1)$

$2x^2 + y^2 = 1$ is given equation of ellipse.

$\Rightarrow 4x + 2yy' = 0$

$\Rightarrow y'|_{(x1,y1)} = -2x_1/y_1$

Therefore, slope of normal at (x_1, y_1) is $y_1/2x_1$

Equation of normal at (x_1, y_1) is

$(y - y_1) = (y_1/2x_1)\ (x - x_1)$

It passes through $(-1/3\sqrt{2}, 0) \Rightarrow -y_1 = y_1/2x_1(-1/3\sqrt{2} - x_1)$

$\Rightarrow x_1 = 1/3\sqrt{2}$

$\Rightarrow y_1 = 2\sqrt{2}/3$

as P lies in first quadrant

Since $(0, \beta)$ lies on the normal of the ellipse at point P, hence we get

$\beta = y_1/2$

$= \sqrt{2}/3$

---

19. If c is a point at which Rolle's theorem holds for the function, f(x) = $log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval [3, 4], where $\alpha \in \mathbf{R}$, then $f''(c)$ is equal to :

a. −1/24
b. −1/12
c. √3/7
d. 1/12

Solution:

Rolle's theorem is applicable on f(x) in [3, 4]

$\Rightarrow$ f(3) = f(4)

$\Rightarrow$ I $In\left(\frac{9+\alpha}{21}\right) = In\left(\frac{16+\alpha}{28}\right)$

$\Rightarrow 9 + \alpha/21 = 16 + \alpha/28$

$\Rightarrow 36 + 4\alpha = 48 + 3\alpha \Rightarrow \alpha = 12$

Now, $f(x) = \ln((x^2+12)/7x) \Rightarrow f'(x) = [7x/(x^2+12)] \times (7x \times 2x - (x^2+12) \times 7)/(7x)^2$

$f'(x) = (x^2 - 12)/(x(x^2 + 12))$

$f'(c) = 0 \Rightarrow c = 2\sqrt{3}$

$f''(x) = (-x^4 + 48x^2 + 144)/x^2(x^2 + 12)^2$

$\therefore f''(c) = 1/12$

---

20. Let $f(x) = x \cos^{-1}(\sin(-|x|))$, $x \in (-\pi/2, \pi/2)$, then which of the following is true ?

a. $f'(0) = -\pi/2$
b. f' is decreasing in $(-\pi/2, 0)$ and increasing in $(0, \pi/2)$
c. f is not differentiable at $x = 0$
d. f' is increasing in $(-\pi/2, 0)$ and decreasing in $(0, \pi/2)$

Solution:

$f(x) = x \cos^{-1}(\sin(-|x|))$

$\Rightarrow f(x) = x \cos^{-1}(-\sin|x|)$

$\Rightarrow f(x) = [\pi - \cos^{-1}(\sin|x|)]$

$\Rightarrow f(x) = x[\pi - (\pi/2 - \sin^{-1}(\sin|x|))]$

$\Rightarrow f(x) = x(\pi/2 + |x|)$

$$\Rightarrow f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right), & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right), & x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \left(\frac{\pi}{2} + 2x\right), & x \geq 0 \\ \left(\frac{\pi}{2} - 2x\right), & x < 0 \end{cases}$$

---

21. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at most three of them are red is.

Solution:

Number of ways to select at most 3 red balls = P(0 red balls) +P(1 red ball) + P(2 red balls) + P(3 red balls)

$= {}^7C_4 + {}^5C_1 \times {}^7C_3 + {}^5C_2 \times {}^7C_2 + {}^5C_3 \times {}^7C_1$

$= 35 + 175 + 210 + 70 = 490$

---

22. Let the normal at a P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at (0,3/2). If m is the slope of the tangent at P to the curve, then $|m|$ is equal to__________.

Solution:

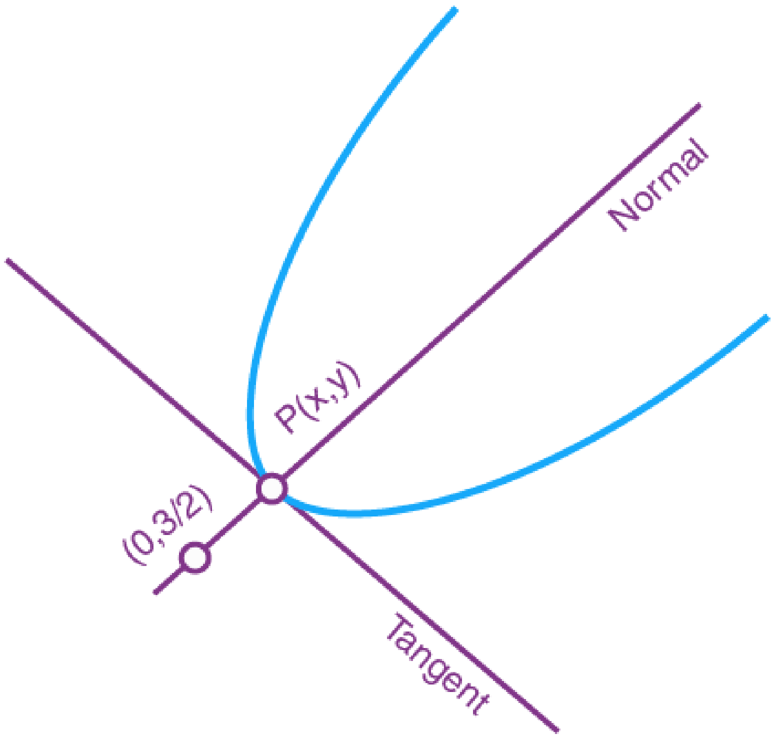
Let co-ordinate of P be (x^1, y^1)

Differentiating the curve w.r.t x

$2yy' - 6x + y' = 0$

Slope of tangent at P

$\Rightarrow y' = 6x_1/(1 + 2y_1)$



m_{normal}=((y_1−3/2)/(x_1−0))

∵ m_{normal}×$m_{tangent}$ = −1

⇒ ((3/2) − y_1/−x_1)×(6x_1/1+2y_1) =−1

⇒y_1=1

⇒x_1=±2

Slope of tangent =±12/3 = ±4

⇒|m|=4

---

23. The least positive value of 'a' for which the equation, 2x^2+(a−10)x+(33/2) =2a has real roots is ___________.

Solution:

∵2x^2+(a−10)x+33/2=2a, a ∈ **Z+** has real roots

⇒D≥0 ⇒(a−10)^2−4×2×((33/2)−2a)≥0

⇒(a−10)^2−4(33−4a)≥0

⇒a^2−4a−32≥0

⇒a∈(−∞,−4]∪[8,∞)

Thus, minimum value of 'a' ∀ a∈**Z+** is 8.

24. The sum $\sum_{k=1}^{20}(1+\ 2+\ 3+\ \ldots\ldots+\ k)$ is__________.

Solution:

$$\sum_{k=1}^{20}\frac{k(k+1)}{2}$$

$$=\ \frac{1}{2}\sum_{k=1}^{20}k^2+\ k$$

$$=\ \frac{1}{2}\left[\frac{20(21)(41)}{6}+\frac{20(21)}{2}\right]$$

=(1/2)[2870+210]=1540

---

25. The number of all 3×3 matrices A, with entries from the set {−1,0,1} such that the sum of the diagonal elements of (AA^{T}) is 3, is _________.

Solution:

Let A = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

And A^{T} = $\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$

(AA^{T})=${a_{11}}^2+{a_{12}}^2+{a_{13}}^2+{a_{21}}^2+{a_{22}}^2+{a_{23}}^2+{a_{31}}^2+{a_{32}}^2+{a_{33}}^2=3$

So out of 9 elements (aij)'s, 3 elements must be equal to 1 or −1 and rest elements must be 0.

So, the total possible cases will be

When there is 6(0's) and 3(1's) then the total possibilities is ${}^9C_6$

For 6(0's) and 3(−1's) total possibilities is ${}^9C_6$

For 6(0's),2(1's) and 1(−1's) total possibilities is ${}^9C_6$ ×3

For 6(0's),1(1's) and 2(−1's) total possibilities is ${}^9C_6$×3

∴ Total number of cases =${}^9C_6$ ×8=672
