JEE (MAIN) 2016

A. General :

1. Immediately fill in the particulars on this page of the Test Booklet with Blue / Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The answer Sheet is kept inside this Test Booklet. When you are directed to pen the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of Chemistry, Mathematics and Physics having total 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
6. Candidates will be awarded marks as stated above in Instructions No. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 6 above.
8. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in one page at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is A. Make sure that the CODE printed on Side-2 of the Answer Sheet and also tally the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. Do not fold or make any stray marks on the Answer Sheet.

PART I – CHEMISTRY

1. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of h/λ (where λ is wavelength associated with electron wave) is given by

(A) 2meV (B) $\sqrt{meV}$ (C) $\sqrt{2meV}$ (D) meV

Ans. (C)

K.E. = eV

$$\lambda \frac{h}{\sqrt{2meV}}$$

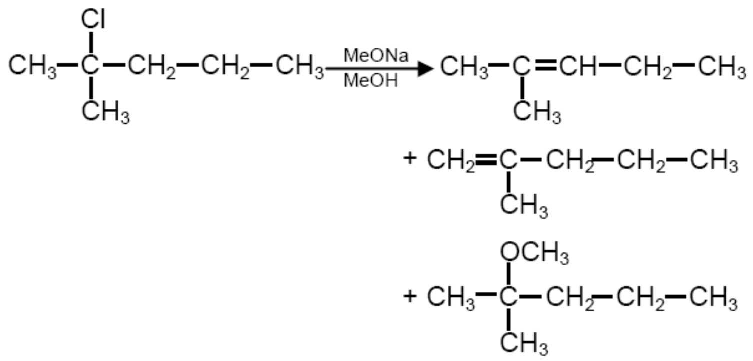
$$\frac{h}{\lambda} = \sqrt{2meV}$$

2. 2-chloro-2-methylpentane on reaction with sodium methoxide in methanol yields :

(i) $C_2H_5CHC(CH_3)_2-OCH_3$ (ii) $C_2H_5CH_2C(CH_3)=CH_2$ (iii) $C_2H_5CH=C(CH_3)-CH_3$

(A) (i) and (iii) (B) (iii) only (C) (i) and (ii) (D) All of these

Ans. (4)



Elimination dominate over substitution in the given reaction but all the products are possible.

3. Which of the following compounds is metallic and ferromagnetic ?

(A) CrO_2 (B) VO_2 (C) MnO_2 (D) TiO_2

Ans. (A)

4. Which of the following statements about low density polythene is FALSE?

(A) It is a poor conductor of electricity.

(B) Its synthesis required dioxygen or a peroxide initiator as a catalyst.

(C) It is used in the manufacture of buckets, dust-bins etc.

(D) Its synthesis requires high pressure.

Ans. (C)

Low density polythene is not used in the manufacturing of buckets, dust-bins etc. because buckets, dustbins are manufactured by high density polythene.

5. For a linear plot of log(x/m) versus log p in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

(A) 1/n appears as the intercept (B) Only 1/n appears as the slope

(C) log (1/n) appears as the intercept (D) Both k and 1/n appear in the slope term

Ans. (B)

According to the Freundlich adsorption isotherm

$$\frac{x}{m} = kP^{1/n}$$

$$\log\frac{x}{m} = \log K + \frac{1}{n}\log P$$

6. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is

(A) 676.5 (B) – 676.5 (C) –110.5 (D) 110.5

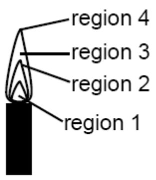
Ans. (C)

$C(s) + O_2(g) \rightarrow CO_2(g)$; $\Delta H = -393.5$ kJ/mol.

$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$; $\Delta H = -283.5$ kJ/mol.

$C(s) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$; $\Delta H = -393.5 + 283.5$ kJ/mol = –110 kJ/mol

7. The hottest region of Bunsen flame shown in the figure is



(A) region 2

(B) region 3

(C) region 4

(D) region 1

Ans. (A)

8. Which of the following is an anionic detergent ?

(A) Sodium lauryl sulphate (B) Cetyltrimethyl ammonium bromide

(C) Glyceryl oleate (D) Sodium stearate

Ans. (A)

Sodium lauryl sulphate = detergent, anionic

Cetyltrimethyl ammonium bromide = detergent, cationic

Glyceryl oleate = detergent, non-ionic

Sodium stearate = soap, anionic

9. 18 g glucosse ($C_6H_{12}O_6$) is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is.

(A) 76.0 (B) 752.4 (C) 759.0 (D) 7.6

Ans. (B)

Moles of glucose = $\frac{18}{180} = 0.1$

Moles of water = $\frac{178.2}{18} = 9.9$

$\Rightarrow N_{Total} = 10$

$\Rightarrow \frac{\Delta P}{P^\circ} = \frac{0.1}{10}$

$\Rightarrow \Delta P = 0.01\,P^\circ$

= 0.01 × 760 = 7.6 torr

P_S = 760 – 7.6

= 752.4 torr

10. The distillation technique most suited for separating glycerol from spent-lye in the soap industry is

(A) Fractional distillation (B) Steam distillation

(C) Distillation under reduced pressure (D) Simple distillation

Ans. (C)

Glycerol is high boiling liquid with B.P. 290°C. It can be separated from spent-lye by distillation under reduced pressure. Liquid is made to boil at lower temperature than normal temperature by lowering pressure on its surface, so external pressure is reduced and B.P. of liquid is lowered hence glycerol is obtained without decomposition at high temperature.

11. The species in which the N atom is in a state of sp hybridization is

(A) NO_2^- (B) NO_3^- (C) NO_2 (D) NO_2^+

Ans. (D)

$NO_2^- = sp^2$

$NO_3^- = sp^2$

$NO_2 = sp^2$

$NO_2^+ = sp$

12. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be

(A) 6.93×10^{-4} mol min^{-1} (B) 2.66 L min^{-1} at STP

(C) 1.34×10^{-2} mol min^{-1} (D) 6.93×10^{-2} mol min^{-1}

Ans. (A)

In 50 minutes, concentration of H_2O_2 becomes $\frac{1}{4}$ of initial.

$\Rightarrow 2 \times t_{1/2} = 50$ minutes

$\Rightarrow t_{1/2} = 25$ minutes

$\Rightarrow K = \frac{0.693}{25}$ pre minutes

$r_{H_2O_2} = \frac{0.693}{25} \times 0.05 = 1.386 \times 10^{-3}$

$2H_2O_2 \rightarrow 2H_2O + O_2$

$r_{O_2} = \frac{1}{2} \times r_{H_2O_2}$

$r_{O_2} = 0.693 \times 10^{-3}$

$r_{O_2} = 6.93 \times 10^{-4}$ mol/minute × litre

13. The pair having the same magnetic moment is : [At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27]

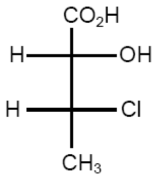
(A) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ (B) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

(C) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$ (D) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

Ans. (A)

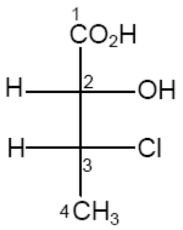
Each $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

Contain 4 unpaired electron.

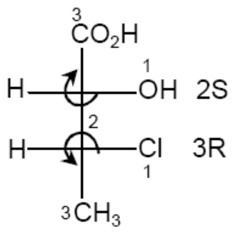
14. The absolute configuration of  is

(A) (2S, 3R) (B) (2S, 3S) (C) (2R, 3R) (D) (2R, 3S)

Ans. (A)



IUPAC numbering



Numbering according to CIP rules for R/S naming

15. The equilibrium constant at 298 K for a reaction A + B $\rightleftharpoons$ C + D is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L–1) will be

(A) 0.818 (B) 1.818 (C) 1.182 (D) 0.182

Ans. (B)

	A	+ B	$\rightleftharpoons$ C	+ D
t = 0	1	1	1	1
t_{eq}	1 – x	1 – x	1 + x	1 + x

$$\Rightarrow \frac{(1+x)^2}{(1-x)^2} = 100 \qquad \Rightarrow \frac{1+x}{1-x} = 10$$

$\Rightarrow$ 1 + x = 10 – 10x $\Rightarrow$ 11x = 9

$$\Rightarrow x = \frac{9}{11}$$

$$\Rightarrow [D] = 1 + \frac{9}{11}$$

$\Rightarrow$ [D] = 1.818

16. Which one of the following ores is best concentrated by froth floatation method?

(A) Siderite (B) Galena (C) Malachite (D) Magnetite

Ans. (B)

Galena = PbS

For sulphur ores froth floatation is carried out.

17. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is

(A) C_3H_8 (B) C_4H_8 (C) C_4H_{10} (D) C_3H_6

Ans. (A)

$$C_xH_y(g) + \left(x + \frac{y}{4}\right)O_2(g) \rightarrow xCO_2(g) + \frac{y}{2}H_2O(\ell) \quad 15\text{ml}$$

$$\text{Volume of } O_2 \text{ used} = \frac{20}{100} \times 375 = 75 \text{ ml}$$

Volume of air remaining = 300 ml

Total volume of gas left after combustion = 330 ml

Volume of CO_2 gases after combustion = 330 – 300 = 30 ml.

$$C_xH_y(g) \quad + \quad \left(x+\frac{y}{4}\right)O_2(g) \quad \rightarrow \quad xCO_2(g) \quad + \quad \frac{y}{2}H_2O(\ell)$$

15 ml 75 ml 30 ml

$$\frac{x}{1}=\frac{30}{15} \quad \Rightarrow \quad x=2$$

$$\frac{x+\frac{y}{4}}{1}=\frac{75}{15} \quad \Rightarrow \quad x+\frac{y}{4}=5$$

$\Rightarrow$ y = 12

$\Rightarrow C_{12}H_{12}$

Confirmed :

Such compound is impossible and also not in option. So it should be bonus.

However if we seriously wish to give an answer then by looking at options, we can see that only C_3H_8 is able to consume 75 ml O_2. So (A) can also be given as answer.

18. The pair in which phosphorous atoms have a formal oxidation state of +3 is:

(A) Pyrophosphorous and hypophosphoric acids

(B) Orthophosphorous and hypophosphoric acids

(C) Pyrophosphorous and pyrophosphoric acids

(D) Orthophosphorous and pyrophosphorous acids

Ans. (D)

Orthophosphorous acid $\left(H_3\overset{+3}{P}O_3\right)$

Pyrophosphorous acid $\left(H_2\overset{+3}{P_2}O_5\right)$

19. Which one of the following complexes shows optical isomerism ?

(A) cis$[Co(en)_2Cl_2]Cl$

(B) trans$[Co(en)_2Cl_2]Cl$

(C) $[Co(NH_3)_4Cl_2]Cl$

(D) $[Co(NH_3)_3Cl_3]$

(en = ethylenediamine)

Ans. (A)

With coordination number six, if two bidentate ligands in cis-position are present, then it is optically active.

20. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces:

(A) NO_2 and NO (B) NO and N_2O (C) NO_2 and N_2O (D) N_2O and NO_2

Ans. (D)

$$Zn + HNO_3(dil.) \rightarrow Zn(NO_3)_2(aq) + N_2O + H_2O$$

$$Zn + HNO_3(conc.) \rightarrow Zn(NO_3)_2 + NO_2 + H_2O$$

21. Which one of the following statements about water is FALSE ?

(A) Water can act both as an acid and as a base.

(B) There is extensive intramolecular hydrogen bonding in the condensed phase.

(C) Ice formed by heavy water sinks in normal water.

(D) Water is oxidized to oxygen during photosynthesis.

Ans. (B)

There is extensive intermolecular hydrogen bonding in the condensed phase.

22. The concentration of fluoride, lead, nitrate and iron in a water sample from an undergroud lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of :

(A) Lead (B) Nitrate (C) Iron (D) Fluoride

Ans. (B)

Highest concentration is of nitrate (100 pm)

23. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively:

(A) LiO_2, Na_2O_2 and K_2O (B) Li_2O_2, Na_2O_2 and KO_2

(C) Li_2O, Na_2O_2 and KO_2 (D) Li_2O, Na_2O and KO_2

Ans. (C)

$$Li + \underset{\text{(excess)}}{O_2(g)} \rightarrow Li_2O$$

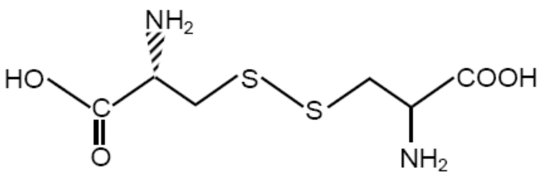
$$Na + \underset{\text{(excess)}}{O_2(g)} \rightarrow Na_2O_2$$

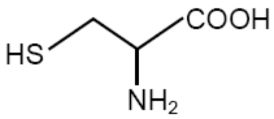
$$K + \underset{\text{(excess)}}{O_2(g)} \rightarrow KO_2$$

24. Thiol group is present in :

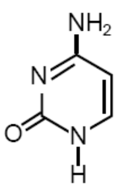
(A) Cystine (B) Cysteine (C) Methionine (D) Cytosine

Ans. (B)

Cystine 

Cysteine  Thiol group (SH) is present in cysteine

Methionine $CH_3-S-CH_2-CH_2-\underset{NH_2}{\underset{|}{C}}H-\overset{O}{\overset{||}{C}}-OH$

Cytosine 

25. Galvanization is applying a coating of :

(A) Cr (B) Cu (C) Zn (D) Pb

Ans. (C)

Galvanization is applying a coating of Zn.

26. Which of the following atoms has the highest first ionization energy ?

(A) Na (B) K (C) Sc (D) Rb

Ans. (C)

I.P_1 = Sc > Na > K > Rb

27. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are :

(A) Four moles of NaOH and two moles of Br_2

(B) Two moles of NaOH and two moles of Br_2

(C) Four moles of NaOH and one mole of Br_2

(D) One mole of NaOH and one mole of Br_2

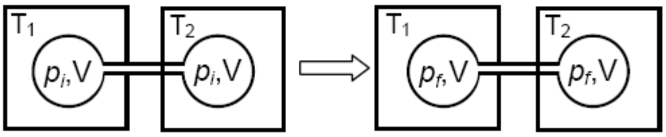
Ans. (C)

Hofmann bromamide degradation reaction

$$R-\overset{\overset{O}{\|}}{C}-NH_2 + Br_2 + 4NaOH \rightarrow R-NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$$

1 mole bromine and 4 moles of NaOH are used for per mole of amine produced.

28. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure pi and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is



(A) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$ (B) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$ (C) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (D) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

Ans. (B)

Initial moles = final moles

$$\frac{P_i \times V}{RT_1} + \frac{P_i \times V}{RT_1} = \frac{P_f \times V}{RT_2} + \frac{P_f \times V}{RT_1}$$

$$\frac{P_i}{T_1} + \frac{P_i}{T_1} = \frac{P_f}{T_2} + \frac{P_f}{T_1}$$

$$\frac{2P_i}{T_1} = P_f\left[\frac{1}{T_2} + \frac{1}{T_1}\right]$$

$$\frac{2P_i}{T_1} = P_f\left[\frac{T_1+T_2}{T_1T_2}\right]$$

$$P_f = 2P_i \times \left(\frac{T_2}{T_1+T_2}\right)$$

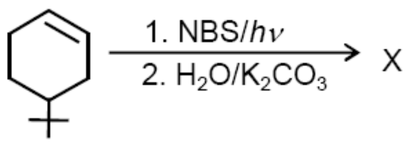
29. The reaction of propene with HOCl (Cl_2 + H_2O) proceeds through the intermediate

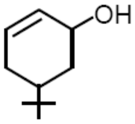
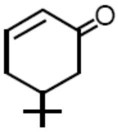
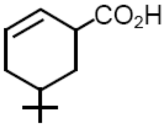
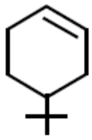
(A) $CH_3 - CH^+ - CH_2 - Cl$ (B) $CH_3 - CH(OH) - CH_2^+$

(C) $CH_3 - CHCl - CH_2^+$ (D) $CH_3 - CH^+ - CH_2 - OH$

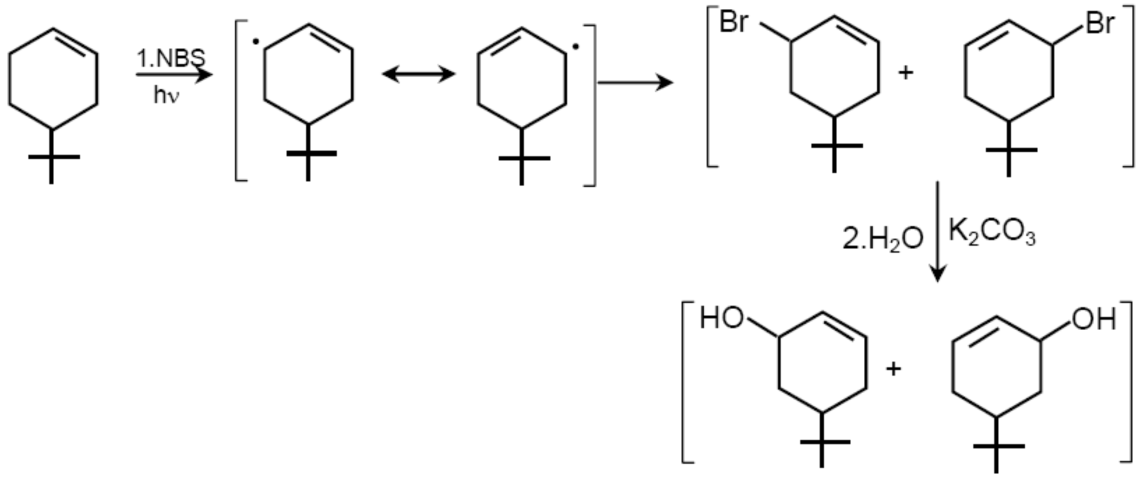
Ans. (A)

30. The product of the reaction give below is



(A)  (B)  (C)  (D) 

Ans. (A)

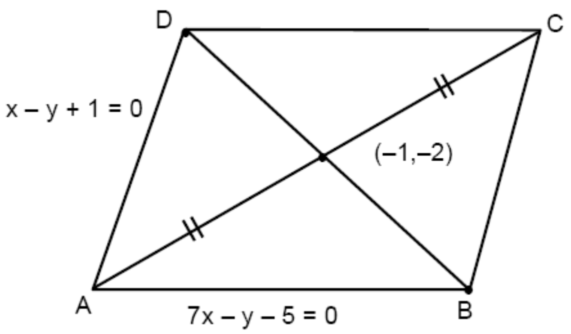


PART II : MATHEMATICS

31. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. if its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus ?

(A) $(-3, -8)$ (B) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (C) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (D) $(-3, -9)$

Ans. (B)



On solving equation of AB & AD

vertex A(1, 2)

$\because$ P is mid point of AC. Hence vertex C is $(-3, -6)$.

So equation of other two sides are $7x - y + 15 = 0$ and $x - y - 3 = 0$.

Hence other vertices are $\left(\frac{1}{3}, -\frac{8}{3}\right)$ and $\left(-\frac{7}{3}, -\frac{4}{3}\right)$

32. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:

(A) $\frac{4}{3}$ (B) 1 (C) $\frac{7}{4}$ (D) $\frac{8}{5}$

Ans. (A)

$a + d, a + 4d, a + 8d \rightarrow$ G.P

$\therefore (a + 4d)^2 = a^2 + 9ad + 8d^2$

$\Rightarrow 8d^2 = ad \Rightarrow a = 8d$

$\therefore 9d, 12d, 16d \rightarrow$ G.P.

common ratio $r = \frac{12}{9} = \frac{4}{3}$

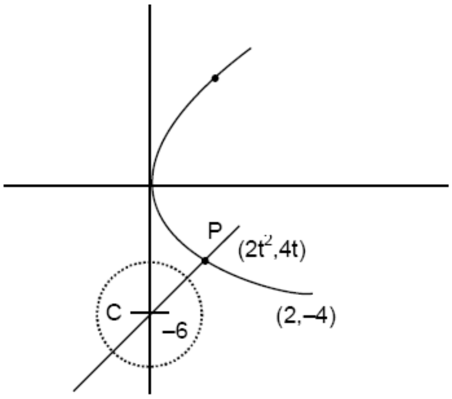
33. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is

(A) $x^2 + y^2 - x + 4y - 12 = 0$ (B) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(C) $x^2 + y^2 - 4x + 9y + 18 = 0$ (D) $x^2 + y^2 - 4x + 8y + 12 = 0$

Ans. (D)

$y = -tx + 2at + at^3$



$-6 = 4t + 2t^3$

$t^3 + 2t + 3 = 0$

$(t + 1)\ (t^2 - t + 3) = 0$

$t = -1$

$\Rightarrow (x - 2)^2 + (y + 4)^2 = r^2 = 8$

$4 + 4 = r^2$

$x^2 + y^2 - 4x + 8y + 12 = 0$

34. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for :

(A) Exactly one value of λ (B) Exactly two values of λ

(C) Exactly three values of λ (D) Infinitely many values of λ

Ans. (C)

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$1(\lambda + 1) - \lambda\ (-\lambda^2 + 1) - 1\ (\lambda + 1) = 0$

$\lambda + 1 + \lambda^3 - \lambda - \lambda - 1 = 0$

$\lambda^3 - \lambda = 0$

35. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \neq 0,$ and S = {$x \in R : f(x) = f(-x)$} ; then S

(A) contains exactly one element (B) contains exactly two elements.

(C) contains more than two elements (D) is an empty set

Ans. (B)

$$f(x) + 2f\left(\frac{1}{x}\right) = 3x$$

S : $f(x) = f(-x)$

$\because\ f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ……(1)

$x \to \frac{1}{x}\quad f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ ……(2)

(1) – 2 × (2) $-3f(x) = 3x - \frac{6}{x}$ $f(x) = \frac{2}{x} - x$

Now f(x) = f(–x)

$\therefore\ \frac{2}{x} - x = -\frac{2}{x} + x$ $\frac{4}{x} = 2x$

$\frac{2}{x} = x \quad \Rightarrow\ x = \pm\sqrt{2}$

Exactly two elements

36. Let $p = \lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{1/2x}$ then log p is equal to

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 2

Ans. (B)

$P = \lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{1/2x}$ then log p =

$P = e^{\lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}-1\right)^{1/2x}} = e^{\lim_{x\to 0^+}\frac{\left(\tan\sqrt{x}\right)^2}{2\left(\sqrt{x}\right)^2}} = e^{1/2}$

$\log P = \log e^{1/2} = \frac{1}{2}$

37. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is

(A) $\frac{\pi}{6}$ (B) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (C) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\frac{\pi}{3}$

Ans. (C)

$\frac{2+3i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta}$

2 – 6 $\sin^2\theta$ = 0 (For purely imaginary)

$\sin^2\theta = \frac{1}{3}$

$\sin\theta = \frac{1}{\sqrt{3}}$

$\theta = \sin^{-1}\frac{1}{\sqrt{3}}$

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

(A) $\frac{4}{\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\frac{4}{3}$

Ans. (B)

Given $2b = \frac{1}{2}.(2ae) \Rightarrow b = \frac{ae}{2}$

$\Rightarrow a^2\left(e^2-1\right) = \frac{a^2e^2}{4} \Rightarrow 3e^2 = 4 \quad \Rightarrow e = \frac{2}{\sqrt{3}}$

39. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(A) $3a^2 - 32a + 84 = 0$ (B) $3a^2 - 34a + 91 = 0$

(C) $3a^2 - 23a + 44 = 0$ (D) $3a^2 - 26a + 55 = 0$

Ans. (A)

Standard deviation of numbers 2, 3, a and 11 is 3.5

$$\therefore\ (3.5)^2 = \frac{\sum x_i^2}{4}(\bar{x})^2$$

$$\Rightarrow\ (3.5)^2 = \frac{4+9+a^2+121}{4} - \left(\frac{2+3+a+11}{4}\right)^2$$

on solving, we get $3a^2 - 32a + 84 = 0$

40. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to

(A) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (B) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

(C) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (D) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

where C is an arbitrary constant

Ans. (A)

$$\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$$

$$\int \frac{\left(\frac{2}{x^3}+\frac{5}{x^6}\right)}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

Let $1+\frac{1}{x^2}+\frac{1}{x^5}=t$

$$\frac{dt}{dx}=\frac{-2}{x^3}-\frac{5}{x^6}$$

$$\int \frac{-dt}{t^3}=\frac{1}{2t^2}+C=\frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C=\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$$

41. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to

(A) 18 (B) 5 (C) 2 (D) 26

Ans. (C)

(i) (3, – 2, –4) lies on the plane

$\therefore\ 3\ell - 2m + 4 = 9 \Rightarrow 3\ell - 2m = 5$ ……(i)

(ii) $2\ell - m - 3 = 0 \Rightarrow 2\ell - m = 3$ …….(ii)

from (i) and (ii)

$\ell = 1$ and $m = -1$

42. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation cosx + cos2x + cos3x + cos4x = 0, is

(A) 5 (B) 7 (C) 9 (D) 3

Ans. (B)

$0 \le x < 2\pi$

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$(\cos x + \cos 4x) + (\cos 2x + \cos 3x) = 0$

$$2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$$

$$2\cos\frac{5x}{2}\left[2\cos x\cos\frac{x}{2}\right] = 0$$

$$\cos\frac{5x}{2} = 0 \text{ or } \cos x = 0 \text{ or } \cos\frac{x}{2} = 0$$

$$x = \frac{(2n+1)\pi}{5} \text{ or } x = (2n+1)\frac{\pi}{2} \text{ or } x = (2n+1)\pi$$

$$x = \left\{\frac{\pi}{3}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$$

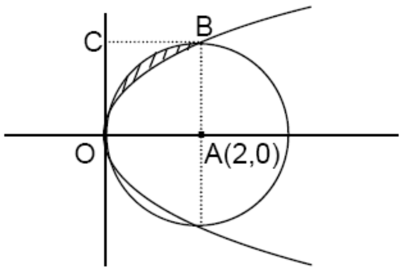
Number of solution is 7

43. The area (in sq.units) of the region $\{(x,y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is

(A) $\pi - \frac{8}{3}$ (B) $\pi - \frac{4\sqrt{2}}{3}$ (C) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (D) $\pi - \frac{4}{3}$

Ans. (A)

$y^2 = 2x$ and $x^2 + y^2 = 4x$ meet at O(0, 0) and B(2, 2) {(2, –2) is not considered as x, y $\ge$ 0}



Now required area = (Area of quadrant of circle) – $\frac{2}{3}$(Area of rectangle OABC)

$$= \pi - \frac{2}{3}.(2.2) = \pi - \frac{8}{3}$$

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is to parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Ans. (C)

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$$

$$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$$

Hence $\vec{a}.\vec{c} = \frac{\sqrt{3}}{2}$ and $\vec{a}.\vec{b} = -\frac{\sqrt{3}}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = \frac{5\pi}{6}$

45. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then

(A) $(4 - \pi)\, x = \pi r$ (B) $x = 2r$ (C) $2x = r$ (D) $2x = (\pi + 4)\, r$

Ans. (B)

$4x + 2\pi r = 2$ .....(i)

$x^2 + \pi r^2$ = minimum $\Rightarrow$ So, $f(r) = \left(\frac{1-\pi r}{2}\right)^2 + \pi r^2$

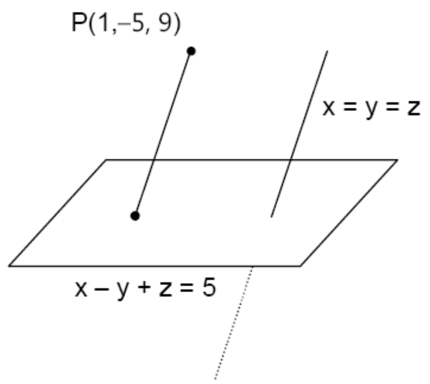
$\frac{df}{dr} = \pi^2 \frac{r}{2} - \frac{\pi}{2} + 2\pi r = 0 \Rightarrow r = \frac{1}{\pi + 4}$

using equation (i) $x = \frac{(1-\pi r)}{2}$

46. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is

(A) $10\sqrt{3}$ (B) $\frac{10}{\sqrt{3}}$ (C) $\frac{20}{3}$ (D) $3\sqrt{10}$

Ans. (A)



Equation of line PQ : $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

$\therefore$ Q can be taken as $(\lambda + 1, \lambda - 5, \lambda + 9)$

As Q lies on plane $x - y + z = 5$

$\therefore (\lambda + 1) - (\lambda - 5) + (\lambda + 9) = 5$

$\lambda = -10 \Rightarrow Q(-9, -15, -1)$

$\therefore$ Required distance PQ = $\sqrt{(1+9)^2 + (-5+15)^2 + (9+1)^2} = \sqrt{100+100+100} = 10\sqrt{3}$

47. If a curve $y = f(x)$ passes through the point (1, –1) and satisfies the differential equation, $y(1 + xy)\, dx = x dy$, then $f\left(-\frac{1}{2}\right)$ is equal to

(A) $-\frac{4}{5}$ (B) $\frac{2}{5}$ (C) $\frac{4}{5}$ (D) $-\frac{2}{5}$

Ans. (C)

y(1 + xy) dx = xdy

ydx – xdy + xy^2dx = 0

$$y^2 d\left(\frac{x}{y}\right) + xy^2 dx = 0$$

$$\frac{x}{y} + \frac{x^2}{2} = C \qquad \ldots\ldots(i)$$

(1, –1) satisfies

$$-1 + \frac{1}{2} = C \Rightarrow C = -\frac{1}{2}$$

Put in (i) $x = -\frac{1}{2}$

$$\frac{-\frac{1}{2}}{y} + \frac{\frac{1}{4}}{2} = -\frac{1}{2} \qquad \Rightarrow \qquad \frac{-1}{2y} = \frac{-1}{2} - \frac{1}{8}$$

$$\frac{1}{2y} = \frac{5}{8}$$

$$y = \frac{4}{5}$$

48. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, x ≠ 0, is 28, then the sum of the coefficients of all the term in this expansion, is

(A) 2187 (B) 243 (C) 729 (D) 64

Ans. (C)

Theoretically the number of terms are 2N + 1 (i.e. odd) But As the number of terms being odd hence considering that number clubbing of terms is done hence the solutions follows

Number of terms = ${}^{n+2}C_2$ = 28 ∴ n = 6

sum of coefficient = $3^n = 3^6 = 729$

put x = 1

49. Consider f(x) = $\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to y = f(x) at $x = \frac{\pi}{6}$ also passes through the point:

(A) $\left(0, \frac{2\pi}{3}\right)$ (B) $\left(\frac{\pi}{6}, 0\right)$ (C) $\left(\frac{\pi}{4}, 0\right)$ (D) (0, 0)

Ans. (A)

at $x = \frac{\pi}{6} \quad \Rightarrow \quad y = \frac{\pi}{3}$

$$f(x) = \tan^{-1}\left(\left|\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right|\right) \qquad \because \ x \in \left(0, \frac{\pi}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$$

$$f(x)=\frac{\pi}{4}+\frac{x}{2} \qquad f'(x)=\frac{1}{2}$$

slope of normal = –2

equation of normal $y-\frac{\pi}{3}=-2\left(x-\frac{\pi}{6}\right)$

$$y=-2x+\frac{2\pi}{3}$$

50. For $x \in R$, f(x) = |log2 – sinx| and g(x) = f(f(x)), then

(A) $g'(0)=\cos(\log 2)$ (B) $g'(0)=-\cos(\log 2)$

(C) g is differentiable at x = 0 and $g'(0)=-\sin(\log 2)$

(D) g is not differentiable at x = 0

Ans. (A)

$$f(x)=|\ell n2-\sin x|$$

$$f(f(x))=|\ell n2-\sin|\ell n2-\sin x$$

In the vicinity of x = 0

$$g(x)=\ell n2-\sin(\ell n2-\sin x)$$

Hence g(x) is differentiable at x = 0 as it is sum and composite of differentiable function

$$g'(x)=\cos(\ell n2-\sin x).\cos x$$

$$g'(x)=\cos(\ell n2)$$

51. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT True ?

(A) E_2 and E_3 are independent (B) E_1 and E_3 are independent

(C) E_1, E_2 and E_3 are independent (D) E_1 and E_2 are independent

Ans. (C)

E_1 : {(4, 1) ,............... (4,6,)} 6 cases

E_2 : {(1,2), ... (6,2)} 6 cases

E_3 : 18 cases (sum of both are odd)}

$$P(E_1)=\frac{6}{36}=\frac{1}{6}=P(E_2)$$

$$P(E_3)=\frac{18}{36}=\frac{1}{2}$$

$$P(E_1\cap E_2)=\frac{1}{36}$$

$$P(E_2\cap E_3)=\frac{1}{12}$$

$$P(E_3\cap E_1)=\frac{1}{12}$$

$$P(E_1\cap E_2\cap E_3)=0$$

$\therefore$ E_1, E_2, E_3 are not independent

52. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A adj A = AA^T, then 5a +b is equal to

(A) 5 (B) 4 (C) 13 (D) – 1

Ans. (A)

$|A|I = AA^T$

$\Rightarrow (10a+3b)\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}5a & -b\\3 & 2\end{bmatrix}\begin{bmatrix}5a & 3\\-b & 2\end{bmatrix}$

$\Rightarrow 25a^2 + b^2 = 10a + 3b$ and $15a - 2b = 0$ and $10a + 3b = 13$

$\Rightarrow 10a + \frac{3.15a}{2} = 13 \Rightarrow 65a = 2 \times 13$

$\Rightarrow a = \frac{2}{5} \Rightarrow 5a = 2$

$\Rightarrow 2b = 6 \Rightarrow b = 3$

$\therefore 5a + b = 5$

53. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to

(A) $p \wedge q$ (B) $p \vee q$ (C) $p \vee \sim q$ (D) $\sim p \wedge q$

Ans. (B)

$[(p \wedge \sim q) \vee q] \vee (\sim p \wedge q)$

$= (p \vee q) \wedge (\sim q \vee q) \vee (\sim p \wedge q)$

$= (p \vee q) \wedge [t \vee (\sim p \wedge q)]$

$= (p \vee q) \wedge t$

$= p \vee q$

54. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is

(A) –4 (B) 6 (C) 5 (D) 3

Ans. (D)

$(x^2 - 5x + 5)^{x^2+4x-60} = 1$

$x^2 - 5x + 5 = 1$ $x^2 + 4x - 60 = 0$ $x^2 - 5x + 5 = -1$

$x^2 - 5x + 4 = 0$ $x = -10, x = 6$ $x^2 - 5x + 6 = 0$

$x = 1, x = 4$ $x = 2, 3$

at $x = 2$, $x^2 + 4x - 60 = -48$ (even)

$\therefore x = 2$ is valid

at $x = 3$, $x^2 + 4x - 60 = -39$ (odd)

$\therefore x = 3$ is invalid

$x = 1, 2, 4, 6, -10$

55. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on

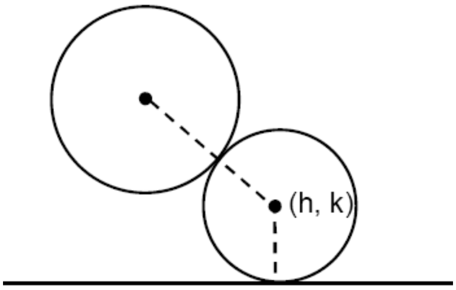
(A) an ellipse which is not a circle (B) a hyperbola

(C) a parabola (D) a circle

Ans. (C)

Parabola

Property : distance from a fixed point & fixed line is equal



56. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is

(A) 59 (B) 52 (C) 58 (D) 46

Ans. (C)

SMALL

A_ _ _ _ # $\frac{4!}{2!}=12$

L_ _ _ _ # 4! = 24

M_ _ _ _ # $\frac{4!}{2!}=12$

SA_ _ _ # $\frac{3!}{2!}=3$

SL_ _ _ # 3! = 6

$\underline{S}\underline{M}\underline{A}\underline{L}\underline{L}$# 1

58th position

57. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)......3n}{n^{2n}}\right)^{1/n}$ is equal to

(A) $\frac{27}{e^2}$ (B) $\frac{9}{e^2}$ (C) 3 log3 – 2 (D) $\frac{18}{e^4}$

Ans. (A)

$$p=\lim_{n\to\infty}\left(\frac{(n+1)(n+2).....(n+2)}{n^{2n}}\right)$$

$$\log p=\frac{1}{n}\left(\lim_{n\to\infty}\sum_{r=1}^{2n}\log\left(1+\frac{r}{n}\right)\right)$$

$$\log p=\int_0^2\log(1+x)dx$$

$$\log p=\left(x\log(1+x)\right)_0^2-\int_0^2\frac{x}{1+x}dx$$

$$\log p=2\log 3-\int_0^2\left(1-\frac{1}{1+x}\right)dx$$

$$\log p=2\log 3-\left(x-\log(1+x)\right)_0^2$$

log p = 2 log3 – (2 – log3)

$$\log p = 3\log 3 - 2 = \log\frac{27}{e^2}$$

$$p = \frac{27}{e^2}$$

58. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots..,$ is $\frac{16}{5}m$, then m is equal to

(A) 101 (B) 100 (C) 99 (D) 102

Ans. (A)

$$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \left(\frac{24}{5}\right)^2 + \ldots. = \frac{8^2}{5^2} + \frac{12^2}{5^2} + \frac{16^2}{5^2} + \frac{20^2}{5^2} + \frac{24^2}{5^2} + \ldots..$$

$$T_n = \frac{(4n+4)^2}{5^2}$$

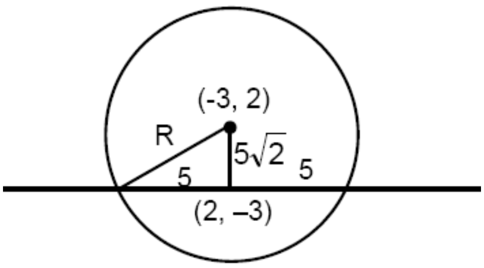
$$S_n = \frac{1}{5^2}\sum_{n=1}^{10} 16(n+1)^2 = \frac{16}{25}\sum_{n=1}^{10}\left(n^2 + 2n + 1\right)$$

$$= \frac{16}{25}\left[\frac{10\times 11\times 21}{6} + \frac{2\times 10\times 11}{2} + 10\right] = \frac{16}{25}\times 505 = \frac{16}{5}m \Rightarrow m = 101$$

59. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (– 3, 2), then the radius of S is

(A) $5\sqrt{3}$ (B) 5 (C) 10 (D) $5\sqrt{2}$

Ans. (A)

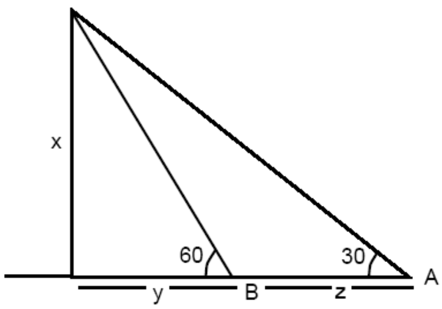


$$r_1 = \sqrt{4+9+12} = 5 \quad \Rightarrow R = \sqrt{25+50} = 5\sqrt{3}$$

60. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°, After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is

(A) 10 (B) 20 (C) 5 (D) 6

Ans. (C)



$$\tan 30^\circ = \frac{x}{y+z} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}x = y + z \;\Rightarrow \tan 60^\circ = \frac{x}{y} = \sqrt{3} \Rightarrow x = \sqrt{3}y = y + z$$

$3y = y + z \quad \Rightarrow 2y = z$

for 2y distance time = 10 min.

so for y dist time = 5 min.

PART III : PHYSICS

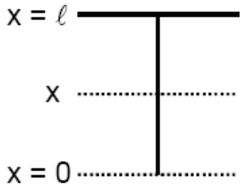
61. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is :

(take g = 10 ms^{-2})

(A) 2 s (B) $2\sqrt{2}$s (C) $\sqrt{2}$s (D) $2\pi\sqrt{2}$s

Ans. (B)

Let mass per unit length be λ.



$T = \lambda g x \quad v = \sqrt{\frac{T}{\lambda}} = \sqrt{gx}$

$v^2 = gx, \ a = \frac{vdv}{dx} = \frac{g}{2}$

$\ell = \frac{1}{2}\frac{g}{2}t^2 \Rightarrow t = \sqrt{\frac{4\ell}{g}} = 2\sqrt{2}\,\text{sec}$

62. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up ? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate.

Take g = 9.8 ms^{-2}

(A) 6.45×10^{-3} kg (B) 9.89×10^{-3} kg (C) 12.89×10^{-3} kg (D) 2.45×10^{-3} kg

Ans. (C)

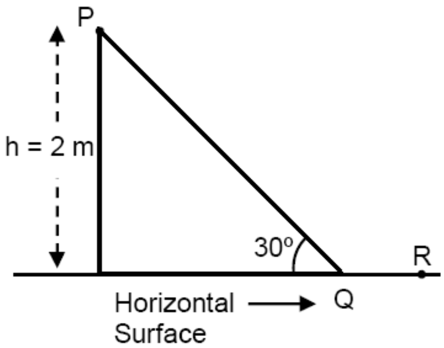
Let m mass of fat is used.

$m\left(3.8\times10^{7}\right)\frac{1}{5} = 10(9.8)(1)(1000)$

$m = \frac{9.8\times5}{3.8\times10^{3}}$

$= 12.89 \times 10^{-3}$ kg

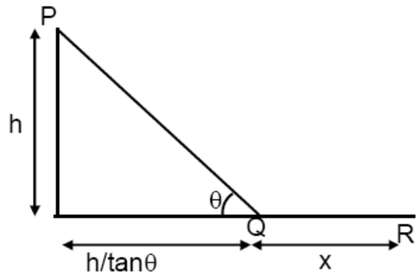
63. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x(= QR), are, respectively close to



(A) 0.2 and 3.5 m (B) 0.29 and 3.5 m

(C) 0.29 and 6.5 m (D) 0.2 and 6.5 m

Ans. (B)



Given that $\frac{\mu mgh}{\tan\theta} = mgh - \frac{\mu mgh}{\tan\theta}$

$\frac{2\mu}{\tan\theta} = 1 \Rightarrow \mu = \frac{\tan\theta}{2}$

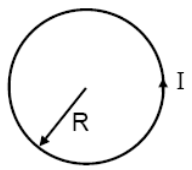
$\mu = 0.29$

$x = \frac{h}{\tan\theta} = 2\sqrt{3} \approx 3.5$ m

64. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

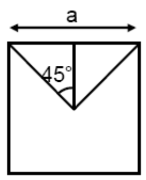
(A) $\frac{\pi^2}{16\sqrt{2}}$ (B) $\frac{\pi^2}{16}$ (C) $\frac{\pi^2}{8\sqrt{2}}$ (D) $\frac{\pi^2}{8}$

Ans. (C)



Magnetic field at centre of circle

$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I \pi}{\ell}$ [Also $\ell = 2\pi R$]



Magnetic field at centre $\frac{4\mu_0 I}{4\pi \frac{a}{2}}(2\sin 45°)$

$= \frac{16\mu_0 I}{\sqrt{2}\pi\ell}$ [Also $4a = \ell$]

Now $\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$

65. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is

(A) 2 Ω (B) 0.1 Ω (C) 3 Ω (D) 0.01 Ω

Ans. (D)

$S = \frac{i_g G}{I - i_g}$

here $i_g = 10^{-3} A \quad G = 10^2 \Omega, \quad I = 10A$

$S \approx 10^{-2} \Omega$

66. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears

(A) 10 times nearer (B) 20 times taller (C) 20 times nearer (D) 10 times taller

Ans. (C)

$\theta = \frac{10}{x}$

$\theta_1 = \frac{10}{x}(20)$

Now 20 times nearer

67. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300 – 400 K, is best described by :

(A) Linear increase for Cu, exponential increase for Si

(B) Linear increase for Cu, exponential decrease for Si

(C) Linear decrease for Cu, linear decrease for Si

(D) Linear increase for Cu, linear increase for Si

Ans. (B)

For conductor (Cu) resistance increases linearly and for semiconductor resistance decreases

Exponentially in given temperature range.

68. Choose the correct statement :

(A) In amplitude modulation the frequency of high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

(B) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(C) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal

(D) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

Ans. (D)

In amplitude modulation amplitude of carrier wave (high frequency) is varied in proportion to the amplitude of signal. In frequency modulation frequency of carrier wave (high frequency) is varied in proportion to amplitude of signal.

69. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively, Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be

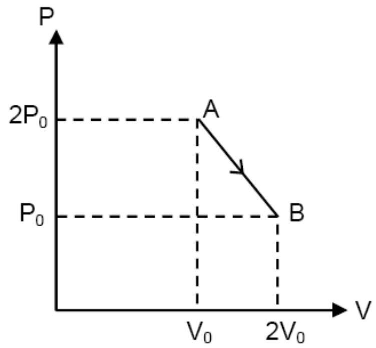
(A) 4 : 1 (B) 1 : 4 (C) 5 : 4 (D) 1 : 16

Ans. (C)

A	B
T_A = 20 min	T_B = 40 min

$$\frac{\left(1-\frac{N}{N_0}\right)_A}{\left(1-\frac{N}{N_0}\right)_B}=\frac{1-\frac{1}{2^{t/t_{1/2}}}}{1-\frac{1}{2^{t/t_{1/2}}}}=\frac{1-\frac{1}{\frac{80}{2^{80}}}}{1-\frac{1}{\frac{80}{2^{80}}}}=\frac{1-\frac{1}{16}}{1-\frac{1}{4}}=\frac{\frac{15}{16}}{\frac{3}{4}}=\frac{5}{4}$$

70. 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be



(A) $\frac{3P_0V_0}{2nR}$ (B) $\frac{9P_0V_0}{2nR}$

(C) $\frac{9P_0V_0}{nR}$ (D) $\frac{9P_0V_0}{4nR}$

Ans. (D)

$$P-P_0=-\frac{P_0}{V_0}(V-2V_0)$$

$$P=3P_0-\frac{P_0}{V_0}V \qquad .....(1)$$

$$\frac{nRT}{V}=3P_0-\frac{P_0}{V_0}V$$

$$nRT=3P_0V-\frac{P_0}{V_0}V^2$$

differentiate w.r.t. Volume

$$3P_0-\frac{2P_0}{V_0}V=0$$

$$V=\frac{3V_0}{2}$$

Put in (1)

$$P=3P_0-\frac{P_0}{V_0}\left(\frac{3V_0}{2}\right)=\frac{3P_0}{2}$$

Now, PV = xRT

$$\frac{9P_0V_0}{4}=nRT$$

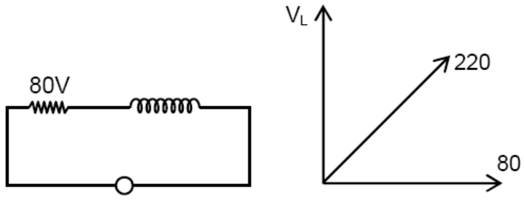
$$T=\frac{9}{4}\frac{P_0V_0}{xR}$$

71. An arc lamp requires a direct current of 10 A at 80 V to function. if it is connected to a 220 V(rms), 50 Hz AC supply, the series inductor needed for it to work is close to

(A) 0.08 H (B) 0.044 H (C) 0.065 H (D) 80 H

Ans. (C)

$$R=\frac{80}{10}=8\Omega$$



$$V_L^2 + 80^2 = 220^2$$

$$V_L^2 = (220+80)(220-80)$$

$= 300 \times 140 \Rightarrow V_L = 204.9$

$I_{ms}\, X_L = 204.9$

$$\frac{220}{\sqrt{64 + x_L^2}} x_L = 2.5$$

72. A pipe open at both ends has fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

(A) $\frac{3f}{4}$ (B) 2f (C) f (D) $\frac{f}{2}$

Ans. (C)

Open organ pipe

$$f = \frac{V}{2\ell} \qquad \text{.....(i)}$$

For closed organ pipe

$$f' = \frac{V}{4\left(\frac{\ell}{2}\right)} = \frac{V}{2\ell} = f$$

73. The box of a pin hole camera, of length L, has hole of radius a. it is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when

(A) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$ (B) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(C) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$ (D) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

Ans. (B)

$b \rightarrow$ radius of spot.

$$b = a + \frac{\lambda L}{a}$$

geometrical spread + spread due to diffraction

$$\frac{db}{da} = 0$$

$$\Rightarrow 1 - \frac{\lambda}{a^2} L = 0$$

$\Rightarrow a^2 = L\lambda$

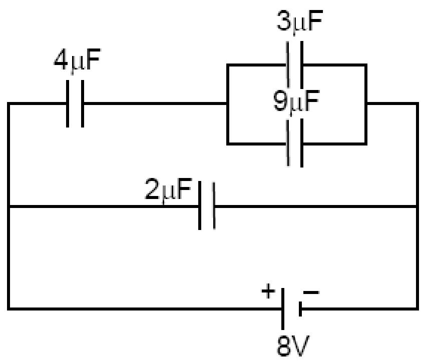
$$\Rightarrow a = \sqrt{L\lambda}$$

$b_{min.} = \sqrt{L\lambda} + \frac{\lambda L}{\sqrt{L\lambda}}$

$b_{min.} = 2\sqrt{L\lambda}$

$b_{min.} = \sqrt{4L\lambda}$

74. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4μF and 9μF capacitors), at a point distance 30 m from it, would equal :



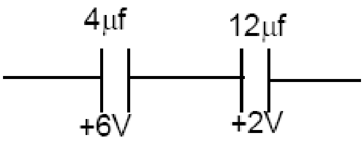
(A) 360 N/C

(B) 420 N/C

(C) 480 N/C

(D) 240 N/C

Ans. (B)



$Q_1 = 24\mu c$

$Q_2 = 18\mu c$

$Q = 42\mu c$

$E = 10^7 \times 42 \times 10^{-6}$

$E = 420$ N/C

75. Arrange the following electromagnetic radiations per quantum in the order of increasing energy :

A : Blue light

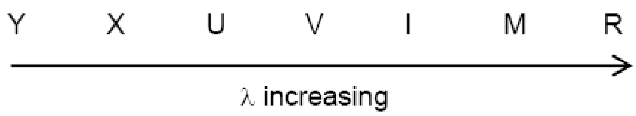
B : Yellow light

C : X-ray

D : Radiowave

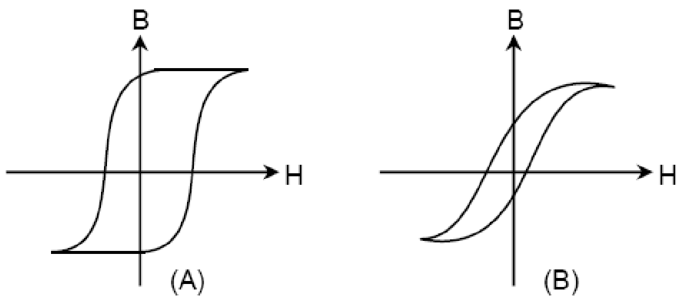
(A) A, B, D, C (B) C, A, B, D (C) B, A, D, C (D) D, B, A, C

Ans. (D)



Hence energy of radio wave will be minimum and maximum for X ray.

76. Hysteresis loops for two magnetic materials A and B are given below :



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use :

(A) A for electromgnets and B for electric generators.

(B) A for transformers and B for electric generators.

(C) B for electromgnets and transformers.

(D) A for electric generators and transformers.

Ans. (C)

Since area of hysterics curve of (B) is smaller it is suitable for electromagnet and transformer.

77. A pendulum clock lose 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively :

(A) 60°C ; $\alpha = 1.85 \times 10^{-4}$/°C (B) 30°C ; $\alpha = 1.85 \times 10^{-3}$/°C

(C) 55°C ; $\alpha = 1.85 \times 10^{-2}$/°C (D) 25°C ; $\alpha = 1.85 \times 10^{-5}$/°C

Ans. (D)

$$\frac{12}{24 \times 3600} = \frac{1}{2}\alpha(40 - T) \quad .....(i)$$

$$\frac{-4}{24 \times 3600} = \frac{1}{2}\alpha(20 - T) \quad .....(ii)$$

from equation (i) and (ii)

$$-3 = \frac{40 - T}{20 - T}$$

– 60 + 3T = 40 – T

4T = 100

T = 25

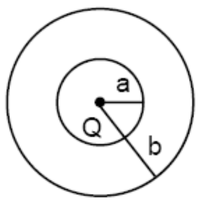
from equation (ii)

$$\frac{-4}{24 \times 3600} = \frac{1}{2}\alpha(20 - 25)$$

$$\frac{4}{24 \times 3600} = \frac{1}{2} \times \alpha \times 5$$

$$\alpha = \frac{8}{24 \times 3600 \times 5} = 1.85 \times 10^{-5} \text{ / °C}$$

78. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is :



(A) $\frac{Q}{2\pi(b^2 - a^2)}$ (B) $\frac{2Q}{\pi(a^2 - b^2)}$ (C) $\frac{2Q}{\pi a^2}$ (D) $\frac{Q}{2\pi a^2}$

Ans. (D)

$$(E)\left(4\pi r^2\right) = \frac{Q + \int_a^r \frac{A}{r} 4\pi r^2 dt}{\varepsilon_0}$$

$$\Rightarrow (E)4\pi r^2 = \frac{Q + \frac{4\pi A}{2}\left(r^2 - a^2\right)}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{A}{\varepsilon_0 2r^2}\left(r^2 - a^2\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{A}{2\varepsilon_0} - \frac{Aa^2}{2\varepsilon_0 r^2}$$

$$\frac{Q}{4\pi\varepsilon_0} = \frac{Aa^2}{2\varepsilon_0}$$

$$A = \frac{Q}{2\pi a^2}$$

79. In an experiment for determination of refractive index of glass of a prism by i –δ, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index ?

(A) 1.6 (B) 1.7 (C) 1.8 (D) 1.5

Ans. (D)

When i = 35° and e = 79° then δ = 40°

δ = i + e – A

40° = 35 + 79 – A

A = 74°

Since i ≠ e so δ_{min} will less than 40°

$$n = \frac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$n = \frac{\sin\left(\frac{40° + 74}{2}\right)}{\sin\left(\frac{74}{2}\right)} = \frac{\sin(57°)}{\sin(37°)} = \frac{0.84}{0.60} = 1.4$$

Since δ_{min} will be less than 40° so

n will be less than 1.4

so the closest answer will be 1.5

80. A student measures the time period of 100 oscillations of a simple pendulum four times. That data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be :

(A) 92 ± 5.0 s (B) 92 ± 1.8 s (C) 92 ± 3 s (D) 92 ± s

Ans. (D)

$$t_{mean} = \frac{90 + 91 + 95 + 92}{4} = 92\,\text{sec.}$$

absolute error in each reading = 2, 1, 3, 0

mean error $= \frac{2 + 1 + 3 + 0}{2} = 1.5$ sec.

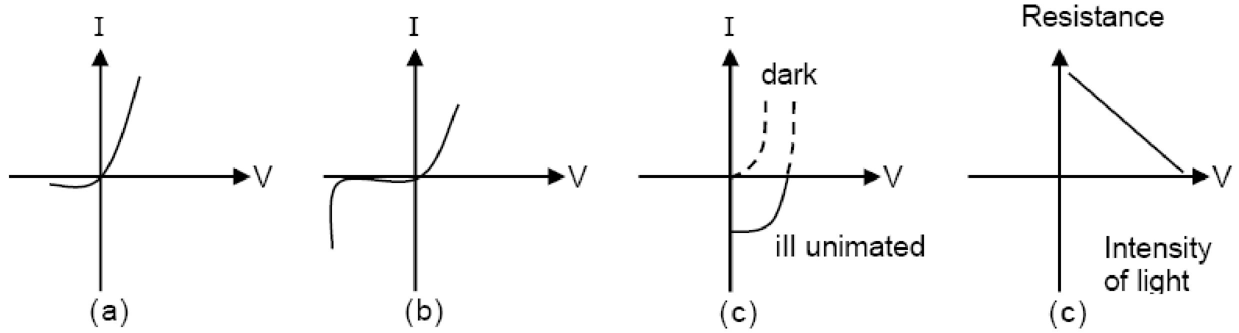
put the least count of the measuring clock is 1 sec.

so it cannot measure upto 0.5 second so we have to round it off.

so mean error will be 2 second

so t = 92 ± 2 sec.

81. Identify the semiconductor devices whose characteristics are given below, in the order (a),(b),(c),(d)



(A) zener diode, simple diode, Light dependent resistance, Solar cell

(B) solar cell, Light dependent resistance, Zener diode, simple diode

(C) zener diode, Solar cell, Simple diode, Light dependent resistance

(D) Simple diode, Zener diode, Solar cell, Light dependent resistance

Ans. (D)

82. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed υ. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be

(A) $<\upsilon\left(\frac{4}{3}\right)^{1/2}$ (B) $=\upsilon\left(\frac{4}{3}\right)^{1/2}$ (C) $=\upsilon\left(\frac{3}{4}\right)^{1/2}$ (D) $>\upsilon\left(\frac{4}{3}\right)^{1/2}$

Ans. (D)

$$\frac{hc}{\lambda}=w+\frac{1}{2}mv^2 \quad .....(i)$$

$$\frac{hc}{\lambda'}=w+\frac{1}{2}m(v')^2$$

$$\frac{hc}{\left(\frac{3\lambda}{4}\right)}=w+\frac{1}{2}m(v')^2 \quad ....(ii)$$

equation $\left[(i)\times\frac{4}{3}\right]-(ii)$

$$\frac{4hc}{3\lambda}-\frac{4}{3}\frac{hc}{\lambda}=\frac{4}{3}w+\frac{4}{3}\left(\frac{1}{2}mv^2\right)-w-\frac{1}{2m}(v')^2$$

$$\Rightarrow \frac{4}{3}w+\frac{4}{3}\left(\frac{1}{2}mv^2\right)=w+\frac{1}{2}m(v')^2$$

$$\Rightarrow \frac{1}{2}m(v')^2=\frac{w}{3}+\frac{4}{3}\frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}m(v')^2>\frac{4}{3}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow v'>\sqrt{\frac{4}{3}}v$$

83. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is.

(A) 3A (B) $A\sqrt{3}$ (C) $\frac{7A}{3}$ (D) $\frac{A}{3}\sqrt{41}$

Ans. (C)

$$v = \omega\sqrt{A^2\left(\frac{2A}{3}\right)^2}$$

$$v = \sqrt{5}\,\frac{A\omega}{3}$$

$$v_{new} = 3v = \sqrt{5}A\omega$$

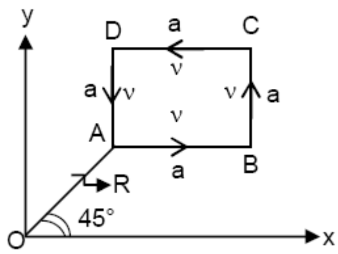
So the new amplitude is given by

$$v_{new} = \omega\sqrt{A_{new}^2 - x^2} \Rightarrow \sqrt{5}A\omega = \omega\sqrt{A_{new}^2 - \left(\frac{2A}{3}\right)^2}$$

$$A_{new} = \frac{7A}{3}$$

84. A particle of mass m is moving along the side of square of side 'a' with a uniform speed υ in the x-y plane as shown in the figure :

Which of the following statements is false for the angular momentum $\vec{L}$ about the origin ?



(A) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D

(B) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

(C) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A

(D) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

Ans. (AC)

From C to D

$$\vec{L}_0 = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$$

from B to C

$$\vec{L}_0 = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$$

from D to A

$$\vec{L}_0 = \frac{mv}{\sqrt{2}}R\left(-\hat{k}\right)$$

from A to B

$$\vec{L}_0 = \frac{mv}{\sqrt{2}}R\left(-\hat{k}\right)$$

85. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_p and C_V are molar specific heat at constant pressure and constant volume, respectively) :

(A) $n = \frac{C - C_p}{C - C_v}$ (B) $n = \frac{C_p - C}{C - C_v}$ (C) $n = \frac{C - C_v}{C - C_p}$ (D) $n = \frac{C_p}{C_v}$

Ans. (A)

$$C = C_v + \frac{R}{1-n}$$

$$C - C_v = \frac{C_p - C_v}{1-n}; \ 1-n = \frac{C_p - C_v}{C - C_v}$$

$$n = 1 - \frac{C_p - C_v}{C - C_v} = \frac{C - C_p}{C - C_v}$$

86. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5mm and the 25^{th} division coincides with the main scale line ?

(A) 0.80 mm (B) 0.70 mm (C) 0.50 mm (D) 0.75 mm

Ans. (A)

When jaws are closed, the zero error will be

= main scale reading + (circularscale reading) (Least count)

= –0.5 mm + (45)(0.01)

zero error = –0.05 mm

when the sheet is placed between the jaws; measured thickness

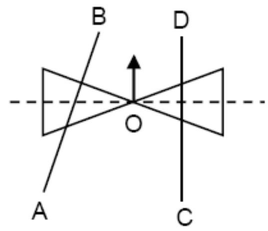
= 0.5 mm + (25)(0.01) = 0.75 mm

$\Rightarrow$ Actual thickness

= 0.75 mm –(–0.05)

= 0.80 mm

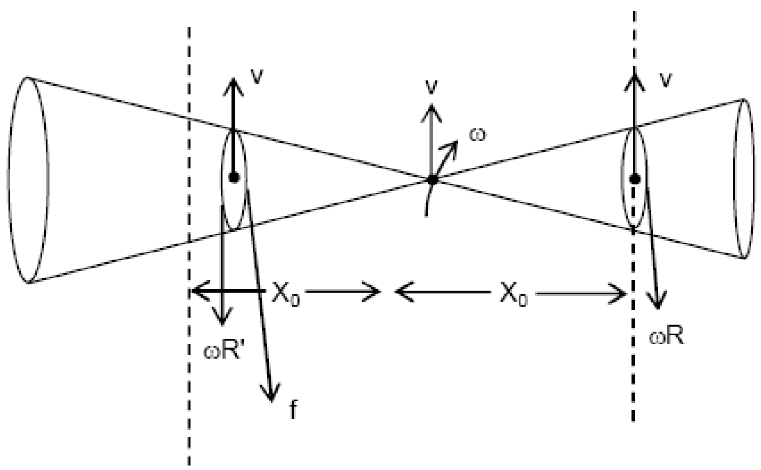
87. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves , the roller will tend to :



(A) turn right (B) go straight

(C) turn left and right alternately (D) turn left

Ans. (D)



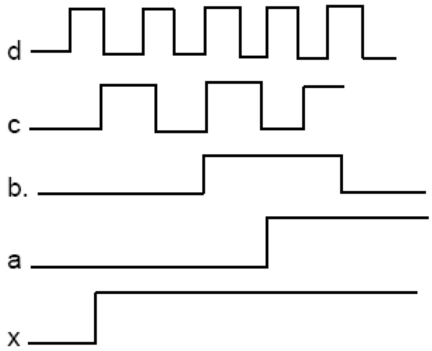
At distance x_0 from O $v = \omega R$

distance less than X_0 $v > \omega R$

Initially, there is pure rolling at both the contacts. As the cone moves forward slipping at AB will start in forward direction as radius at left contact decreases.

Thus the cone will start turning towards left. As it moves further slipping at CD will start in backward direction which will also turn the cone towards left.

88. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is :



(A) AND

(B) OR

(C) NAND

(D) NOT

Ans. (B)

whenever we have 1 at input, output is 1.

so the gate is or

89. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is.

(A) $\alpha = \frac{\beta}{1-\beta}$ (B) $\alpha = \frac{\beta}{1+\beta}$ (C) $\alpha = \frac{\beta^2}{1+\beta^2}$ (D) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

Ans. (AC)

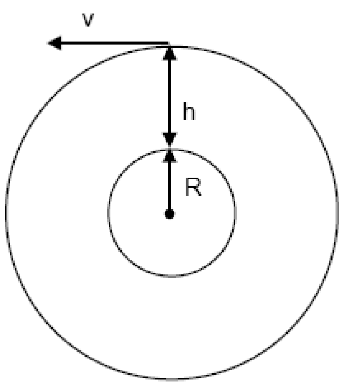
$$\frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\alpha = \frac{\beta}{\beta+1}$$

90. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius or earth R;h<<R). The minimum increase in its orbital velocity required ,so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

(A) $\sqrt{gR}$ (B) $\sqrt{gR/2}$ (C) $\sqrt{gR}\left(\sqrt{2}-1\right)$ (D) $\sqrt{2gR}$

Ans. (C)



$$\frac{GmM}{(R+h)^2} = \frac{GMm}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$

$$\frac{1}{2}mv_1^2 - \frac{GMm}{R} = 0$$

$$v_1 = \sqrt{\frac{2GM}{R}}$$

$$\Delta v = \sqrt{\frac{GM}{R}}\left(\sqrt{2} - 1\right)$$

$$= \sqrt{gR}\left(\sqrt{2} - 1\right)$$