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Knowledge Centre for Success Educate Pvt. Ltd.

for JEE | Aptitude Test | NTSE | Olympiad

JEE MAIN - MODEL PAPER

Time Allotted : 3 hours **M.M.: 360**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This Question Paper contains **30 questions** in each section i.e., **Maths, Physics & Chemistry.**
2. Each page of this question paper has space for rough work. No additional sheets will be provided for rough work.
3. Blank papers, clip boards, log tables, slide rule, calculators, cellular phones, pagers and electronic gadgets, in any form, are **NOT** allowed.
4. The answer sheet, a machine-readable **OMR**, is provided separately.
5. **DO NOT TAMPER WITH/ MUTILATE THE OMR OR THE BOOKLET.**
6. Write your **Name** in the space provided at the bottom of this sheet.

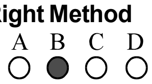
B. Marking Scheme:

7. For each question you will be awarded 4 marks, if you have darkened only the bubble corresponding to the correct answer and zero mark, if none of the bubbles are darkened. In all other cases, minus one (-1) mark will be awarded.
8. There is only one correct response for each question, filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction above.

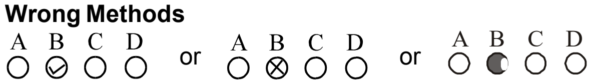
C. Filling the OMR:

9. On the OMR, fill the details and darken the boxes with **BLACK BALLPOINT PEN** only.
10. The answer(s) of the questions must be marked by shading the circles against the question by ballpoint pen only in appropriate manner as shown below. All other method will be treated as wrong.

Right Method



Wrong Methods



Name of the Candidate:

I have read all the instructions and shall abide by them.

Signature of the Candidate

SECTION I : MATHEMATICS

1. $f(x)=\lim_{n\to\infty} x\left[\frac{1}{1.3}+\frac{2}{1.3.5}+\ldots+\frac{n}{1.3.5\ldots(2n+1)}\right]$,

then $\int_0^4 [f(x)]\,d(x-[x])$ (where [.] represent greatest integer function) is equal to

(A) 1 (B) 2
(C) 3 (D) none of these

2. $f(n)=\sum_{r=1}^{n}\left[r^2\left({}^nC_r-{}^nC_{r-1}\right)+(2r+1)\,{}^nC_r\right]$, then f(30) is

(A) 900 (B) 930
(C) 960 (D) none of these

3. Locus of the centre of the circle touching the angle bisectors of the pair of lines $\alpha x^2+\alpha y^2+\beta xy=0$ (where α, β, $\in$ R) is

(A) xy = 0 (B) $x^2 - y^2 = 0$
(C) $x^2 - y^2 = 1$ (D) none of these

4. Statement-1: For any real number θ, $\sin^6\theta + \cos^6\theta \le 1$.
Statement-2: If x and y are non-zero real numbers such that $x^2 + y^2 = 1$, then $x^4 \le 1/2$ and $y^4 \le 1/2$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

5. In an acute angled triangle ABC, the value of $\frac{\sin A}{A}+\frac{\sin B}{B}+\frac{\sin C}{C}$ is

(A) greater than $6/\pi$ (B) $6/\pi$
(C) less than $6/\pi$ (D) none of these

6. Let $f(x)=(x^n-1)^{1/n}\ (x>0)$, then $\int \left(f^6(x)\right)^n dx$, where $f^n(x)$ denotes $\underset{(n\ \text{times})}{fofof....}(x)$, equals

(A) $\frac{x^{n+1}}{n+1}-6x+c$ (B) $\frac{x^n}{n}-64x+c$
(C) $\frac{x^n}{n}-6x+c$ (D) $\frac{x^{n+1}}{n+1}-64x+c$

7. Statement-1: If A(1, 2) and B(7, 10) are two points.
If P (x, y) is such that $\angle APB = 60^o$, then area of ΔAPB will be maximum if P lies on $L \equiv 3x + 4y = 16$
Statement–2: In statement-1 the area of triangle APB is maximum if P lies on the perpendicular bisector of AB.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

8. If the perimeter of triangle is equal to its in-radius then the triangle is necessarily

(A) acute angled
(B) obtuse angled
(C) right angled
(D) no such triangle is possible

9. $\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{1/n}$ equals

(A) e (B) $1/e^3$ (C) e^3 (D) 1/e

10. ABC is a right angle triangle (right angled at B) inscribed in parabola $y^2 = 4x$. The minimum length of the intercept cut off by the tangent at A and C to the parabola on y-axis is

(A) 8 (B) 4
(C) 2 (D) none of these

11. If $\bar{a}, \bar{b}, \bar{c}$ are unit vector equally inclined to each other at an angle $\alpha \ne 0$, then the angle between $\bar{a}\times\bar{b}$ and the plane containing $\bar{b}$ and $\bar{c}$ is

(A) $\sin^{-1}\left(\tan\frac{\alpha}{2}|\cot\alpha|\right)$ (B) $\cos^{-1}\left(\tan\frac{\alpha}{2}|\cot\alpha|\right)$
(C) $\cos^{-1}\left(\cot\frac{\alpha}{2}|\tan\alpha|\right)$ (D) $\sin^{-1}\left(\cot\frac{\alpha}{2}|\tan\alpha|\right)$

12. If $\tan(7x^\circ)=\frac{\sin x^\circ\cos y^\circ+\cos x^\circ\sin y^\circ}{\cos x^\circ\cos y^\circ-\sin x^\circ\sin y^\circ}$ where $x^\circ+y^\circ=134^\circ$, then the least positive integral value of x is

(A) 122 (B) greater than 172
(C) less than 100 (D) 69

13. If A is a square matrix of order 3 such that |A| = 2, then $\left|\left(\text{adj}A^{-1}\right)^{-1}\right|$ is

(A) 2 (B) 4 (C) 8 (D) 16

Space for rough work

14. If $\sum_{n=1}^{49} \frac{1}{\sqrt{n+\sqrt{n^2-1}}} = a + b\sqrt{2}$, then a + b =

(A) 2 (B) 4
(C) 8 (D) none of these

15. If α_1 and α_2 satisfy $\sin^{-1}\frac{2x}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$ where $|\alpha_1 - \alpha_2| < k$, for all α_1 and α_2 then k is equal to

(A) 1 (B) 3/2
(C) 2 (D) none of these

16. Minimum value of $\left((x_1-x_2)^2 + \left(\sqrt{2-x_1^2} - \frac{9}{x_2}\right)^2\right)$, where $x_1 \in (0, \sqrt{2})$ and $x_2 \in R^+$, will be

(A) 4 (B) 6 (C) 8 (D) 10

17. If $y^2 = x^2 - x + 1$ and $I_n = \int \frac{x^n}{y}dx$ and $AI_3 + BI_2 + CI_1 = x^2y$, then ordered triplet (A, B, C) is

(A) $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$ (B) (3, 1, 0)
(C) (1, –1, 2) (D) $\left(3, \frac{-5}{2}, 2\right)$

18. Set of values of λ for which the graph of the function $f(x) = \frac{1}{4}x^4 - \frac{2\lambda}{3}x^3 - 2(\lambda-6)x^2 + 2$ is concave upwards, is

(A) (–2, 2) (B) (–3, 2) (C) (–2, 3) (D) (–6, 3)

19. If the roots of the equation $x^4 - 12x^3 + bx^3 + cx + 81 = 0$ are positive then values of b and c respectively are

(A) 54, 108 (B) –54, 108
(C) 54, –108 (D) none of these

20. If α is an integer satisfying $|\alpha| \le 5 - \|[x]\|$, where x is a real number for which $2x\tan^{-1}x \ge \ln(1+x^2)$ then number of maximum possible values of α will be

(A) 7 (B) 11
(C) 15 (D) none of these

21. With usual notations in triangle ABC, if $A = \frac{\pi}{2}$ and a + b + c = Δ, then b + c – a is equal to

(A) 1 (B) 2 (C) 3 (D) 4

22. If A = {z : |z – 2 – 2i| = |z – 6 – 6i|},
B = {z : arg(z – 4 – 4i) = π/4},
Re(z) = a, Im(z) = b, a $\in$ N, b $\in$ N, a $\le$ 6, b $\le$ 6 then

(A) n(A $\cup$ B) = 7 (B) n(A $\cap$ B) = 1
(C) A $\subseteq$ B (D) B $\subseteq$ A

23. If $\bar{a}, \bar{b}, \bar{c}$ are three non-coplanar unimodular vector, each inclined with other at an angle π/6, then volume of tetrahedron whose edges are $\bar{a}, \bar{b}$ and $\bar{c}$, is

(A) $\frac{\sqrt{3\sqrt{3}-5}}{12}$ (B) $\frac{3\sqrt{3}+5}{12}$
(C) $\frac{3+5\sqrt{2}}{12}$ (D) none of these

24. Number of solutions of equation
$(\sin\theta + \cos\theta)\tan\theta = \sqrt{\sec^2\theta - 1}$ in [0, 2π], are

(A) 1 (B) 2 (C) 3 (D) 4

25. If $S = \sum_{r=1}^{7} \tan^2\frac{r\pi}{16}$, then S is equal to

(A) 44 (B) 40 (C) 34 (D) 35

26. If the curve $y = 1 + \sqrt{4-x^2}$ and the line y = (x – 2) k + 4 has two distinct points of intersection then the range of k is

(A) [1, 3] (B) $\left[\frac{5}{12}, \infty\right)$ (C) $\left(\frac{5}{12}, \frac{3}{4}\right]$ (D) $\left[\frac{5}{12}, \frac{3}{4}\right)$

27. Set of equations $x^2 - b_nx + \underbrace{111\ldots1}_{n-\text{times}} = n$, n = 2, 3, 4, …., 9 and $b_n \in$ N, have integer roots. Which of the following is correct ?

(A) Common root of these equations is 9.
(B) No three of these equations have a common root.
(C) A root of an equation with n = 4 can be 123.
(D) A root of an equation with n = 4 is 1234.

28. A piece of string is cut into two pieces. The point at which the string is cut was chosen at random. What is the probability that the longer piece is at least three times as long as the shorter piece?

(A) 1/4 (B) 1/3 (C) 2/5 (D) 1/2

29. If a_r is the coefficient of x^r in the expression of $(1+x+x^2)^n$, then the value of $a_1 + 4a_4 + 7a_7 + 10a_{10}$ + ... is equal to

(A) 3^{n-1} (B) n 3^{n-1} (C) 2^n (D) $2^n/3$

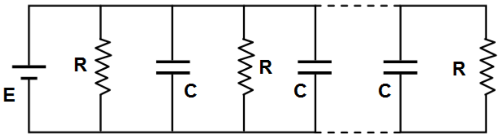
30. Equation of a variable circle is
$x^2 + y^2 - 2(t^2 - 3t + 1)x - 2(t^2 + 2t)y + 2 = 0$.
Locus of centre of circle is

(A) pair of straight line (B) parabola
(C) ellipse (D) none of these

Space for rough work

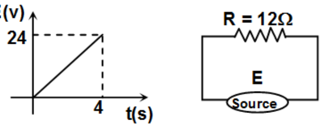
SECTION II : PHYSICS

31. n resistances each of resistance R are joined with capacitors of capacity C (each) and a battery of emf E as shown in the figure. In steady state condition ratio of charge stored in the first and last capacitor is



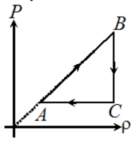
(A) n : 1 (B) (n – 1) : R
(C) $(n^2 + 1) : (n^2 – 1)$ (D) 1 : 1

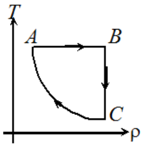
32. A resistance R = 12 Ω is connected across a source of emf as shown in the figure. Its emf changes with time as shown in the graph. What is the heat developed in the resistance in the first four seconds?

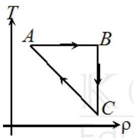


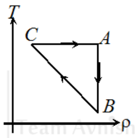
(A) 72 J (B) 64 J (C) 108 J (D) 100 J

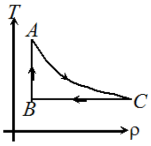
33. Pressure vs density graph of an ideal gas is as shown in the figure.
Then the correct temperature versus density graph is



(A) 

(B) 

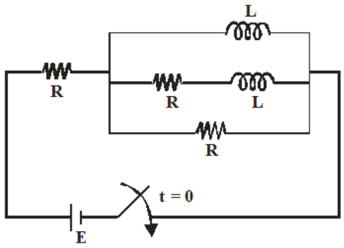
(C) 

(D) 

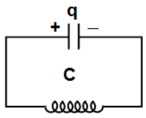
34. An electric field given by $\vec{E} = 4\hat{i} - 3(y^2+2)\hat{j}$ pierces Gaussian cube of side 1m placed at origin such that its three sides represents x, y and z axes. The net charge enclosed within the cube is
(A) $4\varepsilon_0$ (B) $3\varepsilon_0$ (C) $5\varepsilon_0$ (D) zero

35. An electric bulb of 100 W – 300 V is connected with an A.C. supply of 500 V and 150/π Hz. The required inductance to save the electric bulb is
(A) 2 H (B) 1/2 H (C) 4 H (D) 1/4 H

36. Current through e.m.f. sources at t = ∞ will be
(A) E/R
(B) E/3R
(C) E/2R
(D) None of these



37. In an L-C circuit shown in the figure, C = 1F, L = 4H. At time t = 0, charge in the capacitor is 4C and it is decreasing at a rate of $\sqrt{5}$ C/s. Choose the correct statements.



(A) maximum charge in the capacitor can be 6C
(B) maximum charge in the capacitor can be 8C
(C) charge in the capacitor will be maximum after time $2\sin^{-1}(2/3)$ sec
(D) none of these

38. Three infinitely long thin conductors are joined at the origin of coordinates and lies along the x, y and z axes. A current i flowing along the conductor lying along the negative x-axis divides equally into the other two at the origin. The magnitude of magnetic field at the point (0, –a, 0) is
(A) $\frac{\mu_0 i}{4\pi a}$ (B) $\frac{3\mu_0 i}{4\sqrt{2}\pi a}$ (C) $\frac{\sqrt{5}\mu_0 i}{8\pi a}$ (D) $\frac{\sqrt{3}\mu_0 i}{2\pi a}$

39. There are two radioactive nuclei A and B. A is an alpha emitter and B a beta emitter. Their disintegration constants are in the ratio 1 : 2. What should be the ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant
(A) 2 : 1 (B) 1 : 2 (C) e (D) e^{-1}

40. The counting rate observed from a radioactive source at t second was N_0 counts per second and at 4t second $\frac{N_0}{16}$ counts per second. The counting rate observed, as counts per second, at (11/2)t second will be
(A) $\frac{N_0}{128}$ (B) $\frac{N_0}{64}$ (C) $\frac{N_0}{32}$ (D) $\frac{N_0}{256}$

Space for rough work

41. A charge particle is moving in a uniform magnetic field. At any instant magnetic force on particle is $\left(2\hat{i}+4\hat{j}+\hat{k}\right)$ N when its velocity is $\left(\hat{i}+\hat{j}+\lambda\hat{k}\right)$ m/s. The value of λ is

(A) 4 (B) –4 (C) 6 (D) –6

42. In Young's double-slit experiment how many maxima can be obtained on a screen (including the central maximum) on both sides of the central fringe if λ = 2000 Å and d = 7000Å?

(A) 12 (B) 7 (C) 13 (D) 6

43. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column in now

(A) f/2 (B) 3f/4 (C) f (D) 2f

44. A bird is flying over a swimming pool at a height of 2m from the water surface. If the bottom is perfectly plane reflecting surface and depth of swimming pool is 1 m, then the final image of bird from the bird itself is ($\mu_w = 4/3$)

(A) 11/3 m (B) 23/3 m (C) 11/4 m (D) 11/2 m

45. Electrons accelerated by a potential difference of 30 kV fall on a metallic target. The maximum value of momentum of emitted X-ray photons is

(A) 1.6×10^{-22} N-s (B) 1.6×10^{-21} N-s

(C) 1.6×10^{-24} N-s (D) 1.6×10^{-23} N-s

46. If radius or earth were to shrink by one percent, its mass remaining same, the acceleration due to gravity on the earth's surface would

(A) Decrease (B) Remain unchanged

(C) Increase (D) Be zero

47. The current density $\vec{J}$ inside a long, solid cylindrical wire of radius a = 12 mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = \frac{J_0 r}{a}$ where $J_0 = \frac{10^5}{4\pi}\frac{A}{m^2}$. The magnitude of magnetic field at $r = \frac{a}{2}$ in µT is:

(A) 10µT (B) 4µT (C) 5µT (D) 3µT

48. The shortest wavelength of X-rays emitted from an X-ray tube depends on

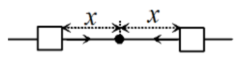
(A) the current in tube

(B) the voltage applied to the tube

(C) the nature of the gas in tube

(D) the atomic number of the target material

49. Two particles are performing simple harmonic motion with same amplitude and same frequency. When they are at same distance from mean position on opposite sides, their speeds are one fourth of maximum speed. Also at these positions their direction of velocities are opposite, the phase difference between the two simple harmonic motions is

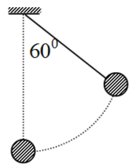


(A) $\pi/2$ (B) π (C) $3\pi/2$ (D) $5\pi/4$

50. A signal of 20 mV is applied to common emitter transistor amplifier circuit. Due to this, the change in the base current and the change in collector current are 20 µA and 2 mA. The load resistance is 10 kΩ. The voltage gain is

(A) 20 V (B) 10 V (C) 50 V (D) 5 V

51. A pendulum of length l = 1 m is released from $\theta_0 = 60^0$. The rate of change of speed of the bob at $\theta = 30^0$ is ($g = 10\ m/s^2$)



(A) $5\sqrt{3}\ m/s^2$

(B) $5\ m/s^2$

(C) $10\ m/s^2$

(D) $2.5\ m/s^2$

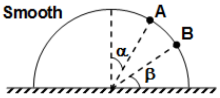
52. A particle is projected with 40 m/sec making angle 60° from horizontal, radius of curvature when it will make angle 30° from horizontal

(A) $\frac{32}{3\sqrt{3}}$ (B) $\frac{32}{\sqrt{3}}$

(C) $\frac{32\sqrt{3}}{3}$ (D) none of these

Space for rough work

53. A particle moves from rest at A on the surface of a smooth circular cylinder of radius r as shown. At B its leaves the cylinder. Then the equation relating α and β is

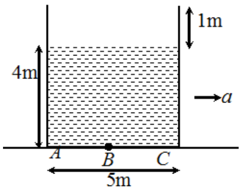


(A) $3\sin\alpha = 2\cos\beta$
(B) $2\sin\alpha = 3\cos\beta$
(C) $3\sin\beta = 2\cos\alpha$
(D) $2\sin\beta = 3\cos\alpha$

54. A particle is moving along x-axis as $x = at^2 + bt + c$; where x is in metres, t in second, a = 1, b = – 3, c = 2, distance covered by particle from t = 1 to t = 2,
(A) 0.5 m (B) zero
(C) 2 m (D) none of these

55. During an adiabatic process, the density of a gas is found to be proportional to cube of temperature. The degree of freedom of gas molecule is
(A) 6 (B) 5 (C) 4 (D) 3

56. A cubical open vessel of side 5m filled with liquid upto a height of 4m is accelerated with an acceleration a. The minimum value of a so that pressure at mid point of AC is equal to atmospheric is



(A) g
(B) 2g
(C) g/2
(D) 2g/5

57. Work done by conservative force when conservative force will be attractive will be negative
(A) always (B) sometime
(C) never (D) none of these

58. If ρ is the density of the material of a wire and σ is the breaking stress. The greatest length of the wire that can hang freely without breaking is
(A) $2\sigma/\rho g$ (B) $\rho/\sigma g$ (C) $\rho g/2\sigma$ (D) $\sigma/\rho g$

59. Two identical spheres are placed on a rough horizontal surface. The sphere A is in pure rolling motion with linear velocity v and B is at rest. If friction is absent between the spheres, long time after elastic collision
(A) A will be at rest and B will move with velocity v
(B) both will have pure rolling motion with linear velocity in same direction
(C) both will move in opposite direction with same speed
(D) both will move in opposite direction with different speed

60. A satellite of mass m, is orbiting around the earth at a distance r from centre of earth, if it stopped suddenly the loss in total energy will be
(A) $\frac{GMm}{2r}$ (B) $-\frac{GMm}{r}$
(C) $-\frac{GMm}{2r}$ (D) none of these

Space for rough work

61. Which of the following statement is incorrect.

(A) Borax can be used as a primary standard for titration against acids .

(B) One mole of borax reacts with two moles of monobasic acid.

(C) Borax can be used as a buffer solution

(D) Borax when dissolved in water gives H_3BO_3 only.

62. Lacttice enthalpy of NaCl(s) is 788 kJ mol^{-1} and enthalpy of hydration is –784 kJ $mole^{-1}$. Hence heat of solution of NaCl(s) is

(A) 4 kJ mol^{-1} (B) –4 kJ mol^{-1}

(C) –1572 kJ mol^{-1} (D) 1572 kJ mol^{-1}

63. For the reaction A(s) $\rightleftharpoons$ B(s) + C(g) K_p = 2 at certain temperature. One mole each of A,B & C are present at equilibrium in a closed container now C(g) is added at equilibrium so that its pressure increased by 2 atm. Then final no of moles at equilibrium of A, B & C respectively are.

(A) 1.5, 0.5 , 0.5 (B) 1.5, 0.1 , 1

(C) 2, 0 , 1 (D) 1.5, 0, 1

64. If $E^0_{Fe^{+2}/Fe}$ is –0.47 , $E^0_{Fe^{+3}/Fe}$ is –0.057 then $E^0_{Fe^{+3}/Fe^{+2}}$ is

(A) 0.77 (B) –0.413 (C) 0.413 (D) –0.77

65. The stability of alkali metal halides are in the order.

(A) NaF < NaCl < NaBr < NaI

(B) NaI < NaBr < NaCl < NaF

(C) NaI < NaBr < NaCl > NaF

(D) None of these

66. The value of change in internal energy (ΔE) when the system absorbs 2000 kJ of heat and does 1200 kJ of work on the surrounding is

(A) 800 kJ (B) –800kJ (C) 3200 kJ (D) –3200 kJ

67. Potential difference of the electrical double layer formed in a calloidal sol is called :

(A) emf (B) Zeta potential

(C) Brownian potential (D) Nerst potential

68. An element crystalises in a bcc lattice nearest neighbours and next nearest neighbours of the element are respectively.

(A) 8,8 (B) 8,6 (C) 6,8 (D) 6,6

69. The vapour pressure of a pure solvent is 100 mm at certain temperature when a non volatile, non electrolyte solute is added vapour pressure drops to 95 mm hence mole fraction of solute in the solution is

(A) 1/20 (B) 1/10 (C) 1/5 (D) 1/4

70. Equivalent conductance at infinite dilution of $BaCl_2$, H_2SO_4 and HCl in aqueous solution are x_1 , x_2 and x_3 respectively. Equivalent conductance of $BaSO_4$ solution is

(A) $x_1 + x_2 – x_3$ (B) $x_1 – x_2 – x_3$

(C) $x_1 + x_2 – 2x_3$ (D) $x_1 – 2x_2 + x_3$

71. Magnetic moment of $[Ti(H_2O)_6]^{3+}$ is

(A) 1.73 BM (B) 2.83 BM (C) 3.87 BM (D) 4.9 BM

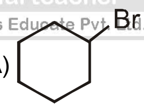
72. Number of possible stereoisomer for complex of type $Ma_2b_2c_2$ are (where a,b,c are monodentate ligands and M is metal ion)

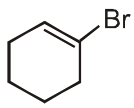
(A) 4 (B) 5 (C) 6 (D) 7

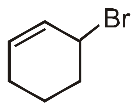
73. Fehling solution consists of two seperate alkaline solution one solution contains $CuSO_4$. The other solution contains.

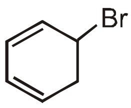
(A) $NaHCO_3$ (B) $KHCO_3$ (C) K_2CO_3 (D) $KNaC_4H_4O_6$

74. Which of the following is dehydrobrominated most easily ?

(A) 

(B) 

(C) 

(D) 

75. Which one of the following oxide is amphoteric in nature

(A) CO (B) CO_2 (C) PbO (D) Bi_2O_3

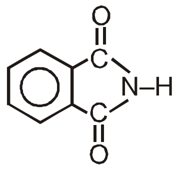
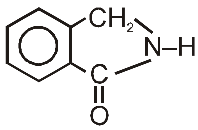
Space for rough work

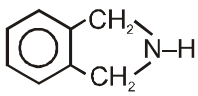
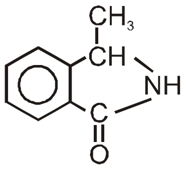
76. Which of the following is a reducing sugar

(A) α–D–Glucose (B) Sucrose

(C) Cellulose (D) All of these

77. Which one of the following contains most acidic H-atom?

(A)  (B) 

(C)  (D) 

78. An alcohol $X(C_4H_{10}O)$ on oxidation with acidic sodium dichromate gives a carboxylic acid $Y(C_4H_8O_2)$. Treatment of X with hot concentrate H_2SO_4 brings about dehydration and gives compound $Z(C_4H_8)$. Treatment of Z with warm aqueous H_2SO_4 gives $W(C_4H_{10}O)$ a new alcohol isomeric with X which is resistant towards oxidation. Identify X.

(A) $CH_3–CH_2–CH_2–CH_2–OH$

(B) $CH_3–CH_2–CH(OH)–CH_3$

(C) $CH_3–CH(CH_3)–CH_2–OH$

(D) $CH_3–C(CH_3)(OH)–CH_3$

79. The reagent used to get 3–phenylprop–2–eoic acid from 4–phenylbut–3–en–2–one is

(A) HIO_4 (B) $KMnO_4$ (C) NaOI (D) $K_2Cr_2O_7$

80. Stability of nitrides of II A group elements is in order:

(A) $Ca_3N_2 < Mg_3N_2 < Be_3N_2$

(B) $Be_3N_2 < Mg_3N_2 < Ca_3N_2$

(C) $Ca_3N_2 < Be_3N_2 < Mg_3N_2$

(D) $Mg_3N_2 < Be_3N_2 < Ca_3N_2$

81. An electron in H-atom in its ground state absorbs 1.5 times as much energy as the minimum required for its escape from the atom. The wave number of the emitted electron is :

(A) $\frac{6.8\ ev}{hc}$ (B) $\frac{13.6\ ev}{hc}$

(C) $\frac{20.4\ ev}{hc}$ (D) none of these

82. The amino acid $H_2N–CH(COOH)–(CH_2)_2–COOH$ at high pH exist as

(A) $H_3\overset{\oplus}{N}–CH(COOH)–(CH_2)_2–COOH$

(B) $H_3\overset{\oplus}{N}–CH(COO^-)–(CH_2)_2–COOH$

(C) $H_3\overset{\oplus}{N}–CH(COO^-)–(CH_2)_2–COO^-$

(D) $H_2N–CH(COO^-)–(CH_2)_2–COO^-$

Space for rough work

83. The correct order of increasing bond enthalpy is

(A) $Cl_2 < F_2 < Br_2 < I_2$ (B) $F_2 < Br_2 < Cl_2 < I_2$

(C) $I_2 < Br_2 < F_2 < Cl_2$ (D) $I_2 < Br_2 < Cl_2 < F_2$

84. Which of the follownig has fractional bond order ?

(A) O_2^{2+} (B) O_2^{2-} (C) F_2^{2-} (D) D_2^{-}

85. To a 0.3 mol of a mixture of $NaHCO_3$ & Na_2CO_3 0.1 mol of NaOH is added. The resulting solution required 0.5 mol of HCl to complete neturalisation, then the composition of initial mixture is

	$NaHCO_3$ (mole)	Na_2CO_3 (mole)
(A)	0.1	0.2
(B)	0.2	0.1
(C)	1.5	1.5
(D)	0	0.3

86. The solubility product of Hg_2Cl_2 at 25°C is 2.0 x 10^{-18} then its solubility in aqueous solution is

(A) $\sqrt{2}$ x 10^{-9} (B) $\frac{1}{2^{\frac{1}{3}}}$ x 10^{-6}

(C) $\frac{(200)^{1/2}}{2}$ x 10^{-5} (D) none of these

87. Number of nuclei of a redioactive substance at time t = 0 are 500 and 400 at time t = 2 sec. The number of nuclei at t = 4 sec will be

(A) 300 (B) 320

(C) 360 (D) none of these

88. Which one is the correct expression below for the solution containing 'n' number of weak acids?

(A) $[H^+] = \sqrt{\sum_{i=1}^{n} \frac{K_i}{C_i}}$ (B) $[H^+] = \sqrt{\sum_{i=1}^{n} K_i C_i}$

(C) $[H^+] = \sum_{i=1}^{n} K_i C_i$ (D) none of these

89. Compute the resonance energy of gaseous benzene from the following data.

B.E(C — H) = 416.3 kJ /mol

B.E(C — C) = 331.4 kJ/ mol

B.E(C = C) = 591.1 kJ/ mol

$$\Delta H^{\circ}_{sub}[C, (graphite)] = 718.4 \text{ kJ/mol}$$

$$\Delta H^{\circ}_{diss}[H_2(g)] = 435.9 \text{ kJ/mol}$$

$$\Delta H^{\circ}_{f}[\text{benzene}(g)] = 82.9 \text{ kJ/mol}$$

(A) –269.7 kJ (B) –138.7 kJ

(C) 104.7 kJ (D) +269.7 kJ

90. At low pressure vander waal's equation for 'n' moles of a real gas will have its simplified from.

(A) $\frac{PV}{RT - \frac{na}{V}} = n$ (B) $\frac{PV}{RT + Rb} = n$

(C) $\frac{PV}{RT - nPb} = 1$ (D) $\frac{PV}{RT - \frac{n^2}{V}} = n$

Space for rough work

ANSWERS

MATHEMATICS

1.	(B)	7.	(D)	13.	(B)	19.	(C)	25.	(D)
2.	(C)	8.	(B)	14.	(C)	20.	(B)	26.	(C)
3.	(A)	9.	(D)	15.	(C)	21.	(D)	27.	(C)
4.	(C)	10.	(B)	16.	(C)	22.	(A)	28.	(D)
5.	(A)	11.	(A)	17.	(D)	23.	(A)	29.	(B)
6.	(A)	12.	(A)	18.	(D)	24.	(C)	30.	(B)

PHYSICS

31.	(D)	37.	(A)	43.	(C)	49.	(B)	55.	(A)
32.	(B)	38.	(C)	44.	(D)	50.	(A)	56.	(B)
33.	(A)	39.	(A)	45.	(D)	51.	(B)	57.	(A)
34.	(B)	40.	(B)	46.	(C)	52.	(D)	58.	(D)
35.	(C)	41.	(D)	47.	(A)	53.	(C)	59.	(B)
36.	(A)	42.	(B)	48.	(B)	54.	(A)	60.	(A)

CHEMISTRY

61.	(D)	67.	(B)	73.	(D)	79.	(C)	85.	(B)
62.	(A)	68.	(B)	74.	(D)	80.	(A)	86.	(B)
63.	(C)	69.	(A)	75.	(C)	81.	(A)	87.	(B)
64.	(A)	70.	(C)	76.	(A)	82.	(D)	88.	(B)
65.	(B)	71.	(A)	77.	(A)	83.	(C)	89.	(A)
66.	(A)	72.	(C)	78.	(C)	84.	(D)	90.	(A)

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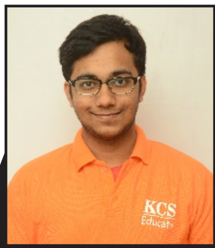

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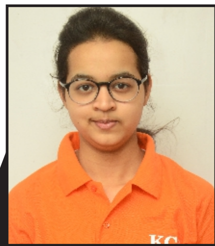

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JEE MAIN - MODEL PAPER

(ANSWERS & HINTS)

SECTION I : MATHEMATICS

1. (B)

$$f(x)=\frac{x}{2}\lim_{n\to\infty}\sum_{r=1}^{n}\left[\frac{1}{1.3.5....(2r-1)}-\frac{1}{1.3.5.....(2r+1)}\right]$$

$$=\frac{x}{2}\lim_{n\to\infty}\left[1-\frac{1}{1.3.5......(2n+1)}\right]=\frac{x}{2}$$

Hence $\int_0^4 [f(x)]d(x-[x])=\int_0^4\left[\frac{x}{2}\right]dx=\int_0^2 0dx+\int_2^4 1dx=2$

2. (C)

$$f(n)=\sum_{r=1}^{n}[r^2({}^nC_r-{}^nC_{r-1})+2r\,{}^nC_r+{}^nC_r]$$

$$=\sum_{r=1}^{n}[(r^2+2r+1)\,{}^nC_r-r^2\,{}^nC_{r-1}]=\sum_{r=1}^{n}[(r+1)^2\,{}^nC_r-r^2\,{}^nC_{r-1}]=\sum_{r=1}^{n}[V_{r+1}-V_r]$$

$$=V_{n+1}-V_1=(n+1)^2\,{}^nC_n-1=n^2+2n$$

3. (A)

The angle bisector of $\alpha x^2+\alpha y^2+\beta xy=0$ is $x^2-y^2=0$ again locus of centre will be angle bisector of $x^2-y^2=0$ hence locus is $xy=0$.

4. (C)

$\sin^6\theta+\cos^6\theta\le 1$

$\Rightarrow(\sin^2\theta+\cos^2\theta)^3-3\cos^2\theta\sin^2\theta(\sin^2\theta+\cos^2\theta)\le 1$

$\Rightarrow 1-3\sin^2\theta\cos^2\theta\le 1$

$\Rightarrow 3\sin^2\theta\cos^2\theta\ge 0$

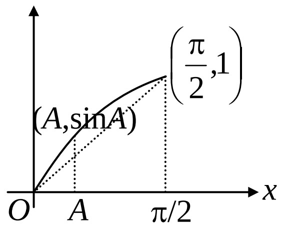
$\Rightarrow\frac{3}{4}(2\sin\theta\cos\theta)^2\ge 0\qquad\Rightarrow\frac{3}{4}(\sin 2\theta)^2\ge 0$

Statement 2 is not necessarily true.

5. (A)

Clearly $\frac{\sin A}{A}>\frac{2}{\pi}$

$\Rightarrow\frac{\sin A}{A}+\frac{\sin B}{B}+\frac{\sin C}{C}>\frac{6}{\pi}$



6. (A)

$f^r(x)=(x^n-r)^{\frac{1}{n}}\quad\Rightarrow\int(f^6(x))^n dx=\int(x^n-6)dx=\frac{x^{n+1}}{n+1}-6x+c$

7. (D)

Since base AB of the triangle in fixed, therefore for maximum area of ΔPAB, height of the triangle should be maximum. This is possible only when P lies on the perpendicular bisector of AB.

Equation of perpendicular bisector of AB is $y-6=-\frac{1-7}{2-10}(x-4)$ or $3x+4y-36=0$.

8. (B)

$2R(\sin A+\sin B+\sin C)=r \quad \Rightarrow \quad \tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}=2$

But we know that $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}\le\frac{1}{3\sqrt{3}}$. Hence no such triangle is possible.

9. (D)

$\ell=\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{1/n} \Rightarrow \log\ell=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\frac{r}{n}=\int_0^1\log x\,dx=-1$

$\Rightarrow \ell=\frac{1}{e}$

10. (B)

Let $A(t_1), B(t_2)$ and $C(t_3)$ be points on the parabola. Since *AB* is perpendicular to BC

$\Rightarrow \frac{4}{(t_1+t_2)(t_2+t_3)}=-1 \qquad \Rightarrow t_2^2+t_2(t_1+t_3)+t_1t_3+4=0$

Discriminant $\ge 0 \Rightarrow (t_1+t_3)^2\ge 4(t_1t_3+4) \qquad \Rightarrow (t_1-t_3)^2\ge 16 \qquad \Rightarrow |t_1-t_3|\ge 4$

$\therefore$ Minimum length of intercept $=|t_1-t_3|=4$

11. (A)

Let required angle is θ

$\therefore \cos(90-\theta)=\left|\frac{(\bar{a}\times\bar{b}).(\bar{b}\times\bar{c})}{|\bar{a}\times\bar{b}||\bar{b}\times\bar{c}|}\right|=\left|\frac{\left((\bar{a}\times\bar{b})\times\bar{b}\right).\bar{c}}{\sin^2\alpha}\right|$

$=\left|\frac{[(\bar{a}.\bar{b})\bar{b}-(\bar{b}.\bar{b})\bar{a}].\bar{c}}{\sin^2\alpha}\right|=\left|\frac{\cos^2\alpha-\cos\alpha}{\sin^2\alpha}\right|=\left|\frac{\cos\alpha.2\sin^2\frac{\alpha}{2}}{\sin\alpha.2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right|=\left|\cot\alpha\tan\frac{\alpha}{2}\right|$

$\therefore \theta=\sin^{-1}\left(\tan\frac{\alpha}{2}|\cot\alpha|\right)$

12. (A)

$\tan(7x^o)=\tan 134^o \Rightarrow 7x=180^\circ n+134 \quad (n\in Z)$

$\Rightarrow x=25n+\frac{1+5n}{7}+19 \quad \Rightarrow$ Least possible value of $n=4$

$\Rightarrow$ Least value of $x=122$

13. (B)

$|\text{Adj } A^{-1}|=|A^{-1}|^2=\frac{1}{|A|^2} \qquad \therefore |(\text{adj } A^{-1})^{-1}|=\frac{1}{|\text{adj } A^{-1}|}=|A|^2=4$

14. (C)

$\because \frac{1}{\sqrt{n+\sqrt{n^2-1}}}=\frac{1}{\sqrt{\left(\sqrt{\frac{n+1}{2}}+\sqrt{\frac{n-1}{2}}\right)^2}}=\frac{1}{\sqrt{\frac{n+1}{2}}+\sqrt{\frac{n-1}{2}}}=\frac{\sqrt{\frac{n+1}{2}}-\sqrt{\frac{n-1}{2}}}{\frac{n+1}{2}-\frac{n-1}{2}}$

$=\sqrt{\frac{n+1}{2}}-\sqrt{\frac{n-1}{2}}$

Hence $a+b\sqrt{2}=\sum_{n=1}^{49}\left(\sqrt{\frac{n+1}{2}}-\sqrt{\frac{n-1}{2}}\right)$

$\Rightarrow a+b\sqrt{2}=\left(\sqrt{\frac{2}{2}}-0\right)+\left(\sqrt{\frac{3}{2}}-\sqrt{\frac{1}{2}}\right)+\left(\sqrt{\frac{4}{2}}-\sqrt{\frac{2}{2}}\right)+\left(\sqrt{\frac{5}{2}}-\sqrt{\frac{3}{2}}\right)+......+\left(\sqrt{\frac{49+1}{2}}-\sqrt{\frac{49-1}{2}}\right)$

$=\sqrt{\frac{49+1}{2}}+\sqrt{\frac{48+1}{2}}-\frac{1}{\sqrt{2}}-0=5+3\sqrt{2} \qquad \Rightarrow a=5,\ b=3$ and $a+b=8.$

15. (C)
since $-1 < x < 1$ hence $-1 < \alpha_1 < 1$ and $-1 < \alpha_2 < 1$ $\Rightarrow |\alpha_1 - \alpha_2| < 2 \Rightarrow k = 2$

16. (C)
Minimum value of $(x_1 - x_2)^2 + \left(\sqrt{2 - x_1^2} - \frac{9}{x_2}\right)^2, x_1 \in (0, \sqrt{2})$ & $x_2 \in R^+$ is equivalent to square of minimum distance between two curves $y = \sqrt{2 - x^2}$ & $y = \frac{9}{x}$

Since minimum distance is along common normal for curve $y = \frac{9}{x} \Rightarrow \frac{dy}{dx} = -\frac{9}{x^2}$

$\therefore$ Normal is $y - y_2 = \frac{x_2^2}{9}(x - x_2)$

This normal passes through (0, 0) [as $y = \sqrt{2 - x^2}$ is semicircle]

$9y_2 = x_2^3 \Rightarrow 9.\frac{9}{x_2} = x_2^3 \Rightarrow x_2 = 3, y_2 = 3$

$\therefore$ Minimum distance $= \sqrt{3^2 + 3^2} - \sqrt{2} = 2\sqrt{2}$

$\therefore$ Minimum value of given expression = $(2\sqrt{2})^2 = 8$

17. (D)
$A\int \frac{x^3}{y} dx + B\int \frac{x^2}{y} dx + C\int \frac{x}{y} dx = x^2 y$

differentiating both sides

$\frac{Ax^3 + Bx^2 + Cx}{y} = 2xy + x^2 \frac{dy}{dx}$

$\Rightarrow Ax^3 + Bx^2 + Cx = 2xy^2 + x^2 y \frac{dy}{dx} = 2x(x^2 - x + 1) + x^2\left(\frac{2x-1}{2}\right) = 3x^3 - \frac{5}{2}x^2 + 2x$

$\Rightarrow A = 3, B = -\frac{5}{2}, C = 2$

18. (D)
$f(x) = \frac{1}{4}x^4 - \frac{2\lambda}{3}x^3 - 2(\lambda - 6)x^2 + 2$ for concave upward, $f''(x) > 0$

$f'(x) = x^3 - 2\lambda x^2 - 4(\lambda - 6)x$ $\Rightarrow f''(x) = 3x^2 - 4\lambda x - 4(\lambda - 6)$

since $f''(x) > 0, \forall x \in R$ $\Rightarrow D < 0$

$\Rightarrow 16\lambda^2 + 4.3.4(\lambda - 6) < 0$ $\Rightarrow \lambda \in (-6, 3)$

19. (C)
Let the roots are x_1, x_2, x_3, x_4

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 12$ and $x_1x_2x_3x_4 = 81$ $\Rightarrow \frac{x_1 + x_2 + x_3 + x_4}{4} = 3$ and $(x_1x_2x_3x_4)^{1/4} = 3$

since A.M. = G.M. $\Rightarrow x_1 = x_2 = x_3 = x_4 = 3$

$\Rightarrow x^4 - 12x^3 + bx^2 + cx + 81 = (x - 3)^4$ $\Rightarrow b = 54, c = -108$

20. (B)
$y = 2x\tan^{-1}x - \ln(1 + x^2)$

$\frac{dy}{dx} = 2\tan^{-1}x$

Since $\frac{dy}{dx} > 0, \forall x \in R^+ \Rightarrow f(x) \geq f(0), \forall x \in [0, \infty)$. Hence $y \geq 0, \forall x \in [0, \infty)$

$\therefore$ Possible value of $5 - |[x]|$ are 0, 1, 2, 3, 4, 5.

Since $|\alpha| \leq 5 - |[x]| \Rightarrow \alpha = 0, \pm1, \pm2, \pm3, \pm4, \pm5$

Hence number of value of α is 11.

21. (D)

$A = \frac{\pi}{2} \Rightarrow b^2 + c^2 = a^2$

$(b + c)^2 - 2bc = a^2$

$(b + c)^2 - a^2 = 2bc = 4\Delta$

$(b + c - a)(b + c + a) = 4(a + b + c)$

$\Rightarrow b + c - a = 4$

22. (A)

$|z - 2 - 2i| = |z - 6 - 6i| \Rightarrow a + b = 8$

$\Rightarrow A = \{(2,6),(3,5),(4,4,),(5,3),(6,2)\}$

$\arg(z - 4 - 4i) = \pi / 4 \quad \Rightarrow a = b \text{ and } a, b > 4 \quad \Rightarrow B = \{(5,5),(6,6)\}$

23. (A)

$$V^2 = \frac{1}{36}[\bar{a}\bar{b}\bar{c}]^2 = \frac{1}{36}\begin{vmatrix} \bar{a}.\bar{a} & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ \bar{b}.\bar{a} & \bar{b}.\bar{b} & \bar{b}.\bar{c} \\ \bar{c}.\bar{a} & \bar{c}.\bar{b} & \bar{c}.\bar{c} \end{vmatrix} \Rightarrow \qquad V = \frac{1}{12}\sqrt{3\sqrt{3} - 5}$$

24. (C)

$(\sin\theta + \cos\theta)\tan\theta = |\tan\theta|$ clearly $\theta = 0, \pi, 2\pi$ are solutions

Case – I: if $\tan\theta > 0$, then $\sin\theta + \cos\theta = 1$

$\therefore \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$

i.e. $\theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$ gives no solution

Case – II: if $\tan\theta < 0$

$\sin\theta + \cos\theta = -1$

$\therefore \sin\left(\theta + \frac{\pi}{4}\right) = -\sin\frac{\pi}{4}$

$\therefore \theta + \frac{\pi}{4} = n\pi + (-1)^n\left(-\frac{\pi}{4}\right)$ gives no solution

$\therefore$ Number of solutions is 3.

25. (D)

$$\tan^2\frac{\pi}{16} + \tan^2\frac{7\pi}{16} + \tan^2\frac{2\pi}{16} + \tan^2\frac{6\pi}{16} + \tan^2\frac{3\pi}{16} + \tan^2\frac{5\pi}{16} + 1$$

$$= \tan^2\frac{\pi}{16} + \cot^2\frac{\pi}{16} + \tan^2\frac{2\pi}{16} + \cot^2\frac{2\pi}{16} + \tan^2\frac{3\pi}{16} + \cot^2\frac{3\pi}{16} + 1$$

$$= \left(\tan\frac{\pi}{16} - \frac{1}{\tan\frac{\pi}{16}}\right)^2 + \left(\tan\frac{2\pi}{16} - \frac{1}{\tan\frac{2\pi}{16}}\right)^2 + \left(\tan\frac{3\pi}{16} - \frac{1}{\tan\frac{3\pi}{16}}\right)^2 + 1 + 6$$

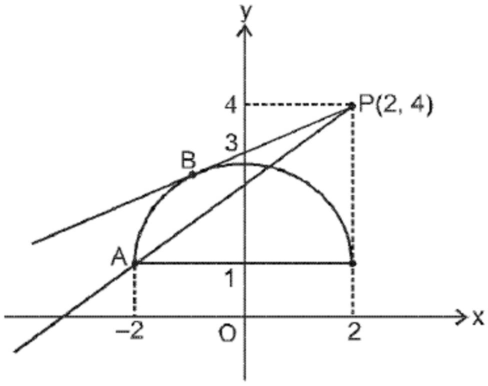
$$= \frac{4}{\tan^2\frac{\pi}{8}} + \frac{4}{\tan^2\frac{2\pi}{8}} + \frac{4}{\tan^2\frac{3\pi}{8}} + 7 = \frac{4}{\tan^2\frac{\pi}{8}} + 4\tan^2\frac{\pi}{8} + 11$$

$$= 4\left(\tan\frac{\pi}{8} - \frac{1}{\tan\frac{\pi}{8}}\right)^2 + 8 + 11 = 4 \times \frac{4}{\tan^2\frac{\pi}{4}} + 19 = 35$$

26. (C)

Line passes through point (2, 4)

slope of PA = $\frac{3}{4}$



slope of $PB = \frac{5}{12}$ $\therefore k \in \left(\frac{5}{12}, \frac{3}{4}\right]$

27. (C)

$x^2 - b_n x + \frac{10^n - 1}{9} = n$

$\Rightarrow x^2 - b_n x + \frac{10^n - 9n - 1}{9} = 0$

Product of roots $\alpha\beta = \frac{10^n - 9n - 1}{9}$

$= \frac{1}{9}\left[{}^nC_0 9^n + {}^nC_1 9^{n-1} + {}^nC_2 9^{n-2} + ... + {}^nC_{n-2} 9^2\right]$

Which is divisible by 9

Hence these equation can have 9 as a common root.

But 9 is not the root for all the equations as $\alpha\beta$ has other divisors, also for example n = 4

$\Rightarrow \alpha\beta = {}^4C_0 9^3 + {}^4C_1 9^2 + {}^4C_2 9$ = 9 × 123

One equation has 123 as a root.

28. (D)

A B C D E

Let AB = BC = CD = DE

The longest piece will be at least 3 times as long as the shortest piece if we cut between A and B or between D and E.

Probability = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

29. (B)

$(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$

$n(1 + x + x^2)^{n-1} (1 + 2x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + ...$

Put x = 1, $n3^n = a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + ...$ (i)

Put $x = \omega$, $0 = a_1 + 2a_2\omega + 3a_3\omega^2 + 4a_4 + 5a_5\omega + 6a_6\omega^2 + ...$ (ii)

Put $x = \omega^2$, $0 = a_1 + 2a_2\omega^2 + 3a_3\omega + 4a_4 + 5a_5\omega^2 + 6a_6\omega + ...$ (iii)

Adding (i), (ii) and (iii) $n3^{n-1} = a_1 + 4a_4 + 7a_7 + 10a_{10} + ...$

30. (B)

Centre of the circle is given by $x = t^2 - 3t + 1$, $y = t^2 + 2t$ eliminating t,

we get $x^2 - 2xy + y^2 - 12x - 13y + 11 = 0$ (the required locus)

SECTION II : PHYSICS

31. (D)

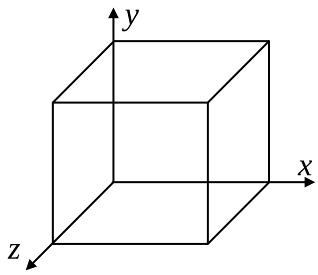
In steady state potential difference across each capacitor = E

32. (B)

33. (A)

AB process is a isothermal, *BC* is isochoric and *CA* is isobaric

34. (B)



Net flux in x-direction = 0

Net flux in y-direction

$= A[3(1)^2 + 2] - A[3(0) + 2]$

$\Rightarrow \frac{q}{\varepsilon_0} = 3A$

$\Rightarrow q = 3\epsilon_0 A = 3\epsilon_0$, (as $A = 1m^2$)

35. (C)

$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P^2} = \frac{300^2}{100} = 900\ \Omega$

$\therefore$ current $i = \frac{V}{R} = \frac{300}{900} = \frac{1}{3}A$

Let L be the required inductance, then

$\frac{500}{z} = \frac{1}{3}$ or $\frac{500}{\sqrt{(900)^2 + x_2^2}} = \frac{1}{3}$

$X_L = 1200\Omega$

$L = \frac{1200}{W} = \frac{1200}{2\pi f} = \frac{1200}{2\pi \times \frac{150}{\pi}} = 4H$

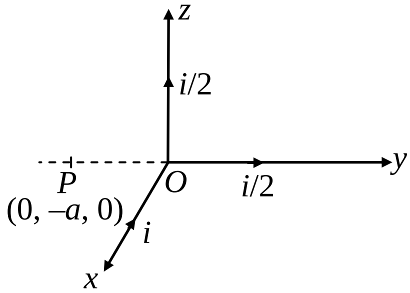
36. (A)

37. (A)

i = $\sqrt{5}$A

$\frac{q_m^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}Li^2 \Rightarrow q_{max} = 6$ C

38. (C)



The field at *P* due to current *i* along x-axis is $\frac{\mu_0 i}{4\pi a}\hat{k}$,

and due to the current $\frac{i}{2}$ along z-axis is $\frac{\mu_0 i}{8\pi a}\hat{i}$.

The resultant field at *P* is $\frac{\mu_0 i}{4\pi a}\sqrt{1^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{8}\frac{\mu_0 i}{\pi a}$

39. (A)

$\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

Probabilities of getting α and β particles are same. Thus rate of disintegration are equal

$\therefore \lambda_A N_A = \lambda_B N_B$ or $\frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = 2$

40. (B)

Here counting rate decrease to $\frac{1}{16}$th of initial in 3*t* second.

So half life time is $T_{\frac{1}{2}} = \frac{3}{4}t$

$\frac{9}{2}t$ corresponds to six half life time $\therefore$ counting rate is $\frac{N_0}{64}$

41. (D)

Force and velocity are perpendicular to each other

So, $\vec{F}.\vec{v} = 0$

$2 + 4 + \lambda = 0$

$\lambda = -6$

42. (B)

$\Delta x = d\sin\theta = n\lambda$, $\sin\theta = \frac{n\lambda}{d}$, $\frac{n7000}{2000} \le 1$, $n \le 3.5$ $n = 3$

Three maximum are formed on both side of central maxima therefore total maxima will be 7.

43. (C)

$f_0 = \frac{v}{2l_0}$ and $f_e = \frac{v}{4l_c}$

Since, tube is half dipped in water, $l_c = \frac{l_0}{2}$ $\therefore$ $f_c = \frac{v}{4\left(\frac{l_0}{2}\right)} = \frac{v}{2l_0} = f_0 = f$

44. (D)

Mirror will look at a distance at $\frac{t}{\mu} = \frac{1}{4/3} = \frac{3}{4}\text{m}$ from water surface and mirror will be at a distance of $\left(2 + \frac{3}{4}\right)\text{m}$ from bird.

Image of bird from bird is $2 \times \frac{11}{4} = \frac{11}{2}\text{m}$

45. (D)

Energy of the electron is 3×10^4 eV

So the maximum momentum of photon is $P = \frac{hv}{c} = \frac{3\times10^4 \times 1.6\times10^{-19}}{3\times10^8}$ = 1.6×10^{-23} N-s

46. (C)

$g = \frac{GM}{R^2}$

$g \propto \frac{1}{R^2}$; of M is same

47. (A)

$\int \vec{B}.d\vec{L} = \mu_0 I$

$B\ 2\pi r = \mu_0 \int \frac{J_0 r}{a} 2\pi r dr$

48. (B)

49. (B)

Let equation of motion of one particle be $x = x_0 \sin \omega t$ and $v = v_0 \cos \omega t$

For the second particle, $x' = x_0 \sin(\omega t + \phi)$ and $v' = v_0 \cos(\omega t + \phi)$ where ϕ is phase difference between the two SHMs.

From question, $x = -x'$ and $v = -v'$

50. (A)

51. (B)

Rate of change of speed $\left(\frac{dv}{dt}\right) = \frac{\text{tangential force}}{\text{mass}} = \frac{mg \sin 30^0}{m} = g \sin 30^0$

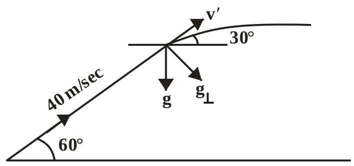
$= 10\left(\frac{1}{2}\right)$ m/s^2 = 5 m/s^2

52. (D)

$40 \cos 60° = v' \cos 30°$

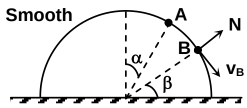
$g_\perp = g \cos 30°$

$40 \times \frac{1}{2} = v' \times \frac{\sqrt{3}}{2}$



$$R = \frac{v'^2}{g \times \frac{\sqrt{3}}{2}} = \frac{(40)^2 \times 2}{3 \times 10 \times \sqrt{3}} = \frac{3200}{3 \times 10 \times \sqrt{3}} = \frac{320}{3\sqrt{3}}$$

53. (C)



At B, mg sin β = $\frac{mv_B^2}{r}$ . . . (1)

Using energy considerations

$\frac{1}{2}mv_B^2$ = mgr(cos α – sin β) . . . (2)

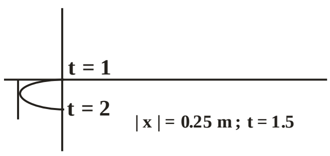
From (1) and (2)

mg sin β = 2mg(cosα – sin β)

2cosα = 3 sin β

54. (A)

$\frac{dx}{dt} = 0$; t = 1.5 sec;



Total distance from t = 1 sec to t = 2 sec

= 0.25 + 0.25

= 0.5 m

55. (A)

For adiabatic process, $TV^{\gamma-1}$ = constant

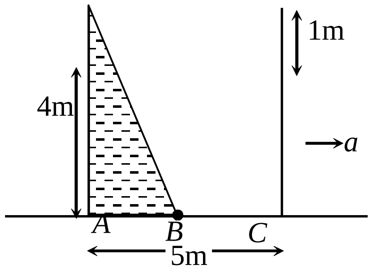
$T\left(\frac{m}{\rho}\right)^{\gamma-1}$ = constant

$\frac{T}{\rho^{\gamma-1}}$ = constant

$\rho \propto T^{1/(\gamma-1)} \Rightarrow \frac{1}{\gamma-1} = 3 \Rightarrow \gamma = 4/3$

$f = \frac{2}{\gamma-1} = \frac{2}{\left(\frac{4}{3}-1\right)} = 6$

56. (B)



Pressure at point *B* will be atmospheric if layer of water will be as shown

Hence $\tan\theta = \frac{5}{2.5} = 2 = \frac{a}{g}$ or a = 2*g*

57. (A)

58. (D)

Maximum stress on the wire will be at highest point (at the point of suspension)

$\therefore$ stress = $\frac{\text{weight}}{A}$ = $\frac{Al\rho g}{A} = \sigma$

$\therefore$ $l = \frac{\sigma}{\rho g}$

59. (B)

Just after collision, *B* will have only linear velocity *v* and *A* will have only angular velocity ω. Long time after collision both will have pure rolling in same direction

60. (A)

$U_{\text{potential}} = -\frac{GMm}{r}$

Kinetic Energy = $\frac{1}{2}m \times \frac{GM}{r} = \frac{GMm}{2r}$

When it will stop ; kinetic energy = 0

SECTION III : CHEMISTRY

61. (D)

62. (A)

63. (C)

$$A(s) \rightleftharpoons B(s) + C(g)$$

At eq. mole 1 1 1, $K_p = 2 = P_c$

So, no. of moles of C(g) will require to increase it's plessure by 2 atom = 1 mole

$$A(s) \rightleftharpoons B(s) + C(g)$$

Before eq. mole 1 1 2

At new eq. mole 2 0 1

64. (A)

65. (B)

66. (A)

67. (B)

68. (B)

69. (A)

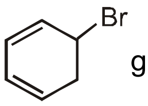
70. (C)

71. (A)

72. (C)

73. (D)

74. (D)


gives most stable product.

75. (C)

76. (A)

α–D–Glucose is a hemiacetal structure.

77. (A)

It is most acidic as electron density present at the N-atom is being attracted by $>C=O$ group present towards both side of imide ($>NH$) group.

78. (C)

$$\text{(X)}\ CH_3-\overset{\overset{CH_3}{|}}{C}H-CH_2-OH \xrightarrow[\Delta]{H_2SO_4} CH_3-\overset{\overset{CH_3}{|}}{C}=CH_2\ \text{(Z)} \xrightarrow{H_2SO_4/H_2O} CH_3-\underset{\underset{OH}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_3\ \text{(W)}$$

(X) $\downarrow$ [O]

$$\text{(Y)}\ CH_3-\overset{\overset{CH_3}{|}}{C}H-COOH$$

79. (C)

It is idoform reaction

$$C_6H_5CH=CH-\overset{\overset{O}{||}}{C}-CH_3 \xrightarrow[\text{Solution}]{\text{NaOI}} \xrightarrow{\text{acidification}} C_6H_5-CH=CH-\overset{\overset{O}{||}}{C}-OH + CHI_3$$

80. (A)

81. (A)

82. (D)
In high pH i.e. basic medium, acidic H is taken by base.

83. (C)

84. (D)

85. (B)
Let x moles of $NaHCO_3$, (0.3 - x) moles of Na_2CO_3

	$NaHCO_3$ +	NaOH →	Na_2CO_3 +	H_2O
BR mole	x	0.1	(0.3 - x)	0
AR mole	0	(0.1 -x)	0.3	x

$Na_2CO_3 + 2HCl \rightarrow 2\,NaCl + CO_2 + H_2O$

$NaOH + HCl \rightarrow NaCl + H_2O$

So, total moles of HCl required w.r.t. Na_2CO_3 and NaOH = 0.3 × 2 + (0.1 - x) = 0.7 - x

$\therefore$ 0.7 - x = 0.5

x = 0.2

86. (B)

87. (B)

$$\lambda = \frac{\ell n\frac{5}{4}}{2} = \frac{\ell n\frac{500}{x}}{4}$$

$$2\ell n\frac{5}{4} = \ell n\frac{500}{x}$$

$$\left(\frac{5}{4}\right)^2 = \frac{500}{x}$$

x = 320

88. (B)

$HA \rightleftharpoons H^+ + A^-$; $K_1 = \frac{[H^+][A^-]}{[HA]}$

$HB \rightleftharpoons H^+ + B^-$; $K_2 = \frac{[H^+][B^-]}{[HB]}$

By mass balance, $[HA]_{initial} = [HA]_{eq} + [A^-] = C_1$, $[HB]_{initial} = [HB]_{eq} + [B^-] = C_2$

By charge balance, $\left[H^+\right] = \left[A^-\right] + \left[B^-\right]$

$$[H^+] = \frac{K_1[HA]}{[H^+]} + \frac{K_2[HB]}{[H^+]}$$

$[H^+]^2 = K_1\,[HA] + K_2\,[HB] = K_1\,\{C_1 - [A^-]\} + K_2\,\{C_2 - [B^-]\}$

If K_1 , K_2 are very less then

$[H^+] = \sqrt{K_1C_1 + K_2C_2 + ...K_nC_n} = \sqrt{\sum_{i=1}^{n} K_i\,C_i}$ for 'n' number of weak acids

89. (A)

To compute resonance energy, we compare the calculated value of ΔH_f° (benzene, g) with the given one. To calculate ΔH_f° (benzene, g), we add the following reactions.

$6C(g) + 6H(g) \rightarrow C_6H_6(g)$ $\qquad \Delta H^\circ = -(3B.E_{C-C} + 3B.E_{C=C} + 6B.E_{C-H})$

$6C(graphite) \rightarrow 6C(g)$ $\qquad \Delta H^\circ = 6 \times 718.4$ kJ/ mol

Add $3H_2(g) \rightarrow 6H(g)$ $\qquad \Delta H^\circ = 3 \times 435.9$ kJ/ mol

$$6C(graphite) + 3H_2(g) \rightarrow C_6H_6(g)$$

The corresponding enthalpy change is

$$\Delta H_f^0 = -(3B.E_{C-C} + 3B.E_{C=C} + 6B.E_{C-H}) + [6 \times 718.4 + 3 \times 435.9] \text{ kJ/mol}$$

$= [-(3 \times 331.4 + 3 \times 591.1 + 6 \times 416.3) + (6 \times 718.4 + 3 \times 435.9)]$ kJ/ mol

ΔH_f^0 = 352.8 kJ/mol

Resonance energy = $\Delta H_f(\text{exp.}) - \Delta H_f(\text{Cal.})$

= 82.9 – 352.8 = –269.7 kJ/mol

90. (A)