JEE ADVANCED-2016 PAPER-1

PART – I: PHYSICS

1. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

λ(μm)	V_0(Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that c = 3 × 10^8 ms^{-1} and e = 1.6 × 10^{-19}C, Planck's constant (in units of J s) found from such an experiment is

(A) 6.0 × 10^{-34} (B) 6.4 × 10^{-34} (C) 6.6 × 10^{-34} (D) 6.8 × 10^{-34}

Ans. (B)

$$eV = \left[\frac{hc}{\lambda} - \phi\right]$$

$$e(V_1 - V_2) = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = \frac{hc(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$$

$$h = \frac{e(V_1 - V_2)\lambda_1\lambda_2}{c(\lambda_2 - \lambda_1)}$$

$$= \frac{1.6 \times 1 \times 0.3 \times 0.4 \times 10^{-12} \times 10^{-19}}{3 \times 10^8 \times 0.1 \times 10^{-6}} = 6.4 \times 10^{-34}.$$

Alternate solution

Slope of V_0 vs $\frac{1}{\lambda} = \frac{hc}{e}$

From first two data points.

Slope = $\left(\frac{2.0 - 1.0}{\frac{1}{0.4} - \frac{1}{0.3}}\right) \times 10^{-6}\,Vm = \frac{hc}{e}$

$$\Rightarrow 1.2 \times 10^{-6} = \frac{h \times 3 \times 10^8}{1.6 \times 10^{-19}}$$

$\Rightarrow$ 6.4 × 10^{-34}.

We get same value from second and third data points.

2. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h (<ℓ) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 300 with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are (g = 10 ms^{-2})

(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$ (B) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}N$

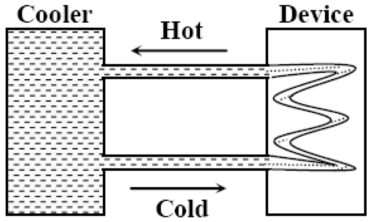
(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}N$ (D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

Ans. None

On the basis of given data, it is not possible for the rod to stay at rest.

Note: If we take the normal reaction at the wall equal to the normal reaction at the floor, the answer will be D.

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30 ^{0}C and the entire stored 120 litres of water is initially cooled to 10 ^{0}C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

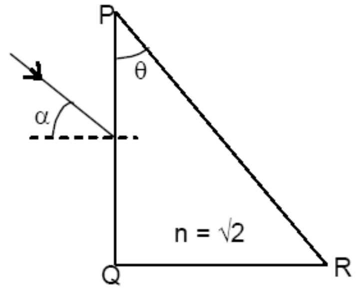
Ans. (B)

$(P_{heater} - P_{cooler}) \times t = ms\Delta T$.

$\Rightarrow (3 \times 10^3 - P) \times 3 \times 3600 = 120 \times 4.2 \times 10^3 \times 20$

$\Rightarrow P = 2067$

4. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45^0. The angle θ of the prism is



(A) 15^0

(B) 22.5^0

(C) 30^0

(D) 450

Ans. (A)

$i = \beta + \theta$

For $\alpha = 45^0$, by Snell's law,

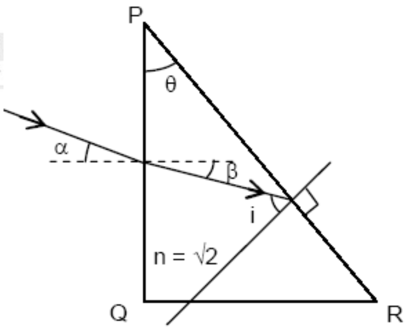
$1 \times \sin 45^0 = \sqrt{2} \sin\beta$

$\Rightarrow \beta = 30^0$

For TIR on face PR,

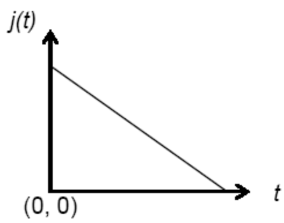
$$\beta + \theta = \theta_c = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^0$$

$\Rightarrow \theta = 45° - \beta = 15^0$.

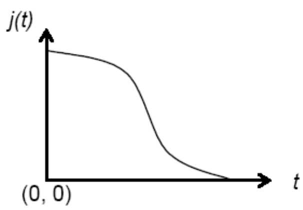


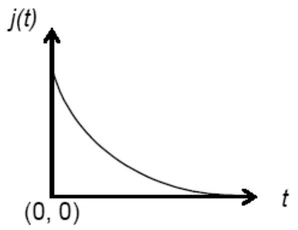
5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

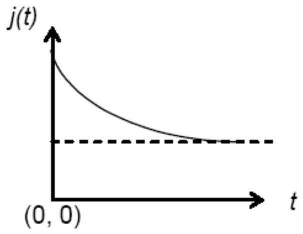
(A)



(B)



(C) 

(D) 

Ans. (C)

Fro infinite line,

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$\Rightarrow dV = \frac{-\lambda}{2\pi\varepsilon r} dr$$

Current through an elemental shell;

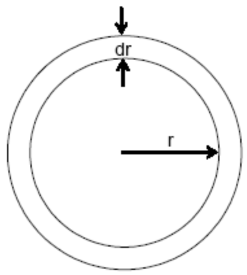
$$I = \frac{|dV|}{dR} = \frac{\frac{\lambda}{2\pi\varepsilon r} dr}{\frac{1}{\sigma} \times \frac{dr}{2\pi r \ell}} = \frac{\lambda\sigma\ell}{\varepsilon}$$

This current is radically outwards so;

$$\frac{d}{dt}(\lambda\ell) = \frac{-\lambda\sigma\ell}{\varepsilon} \Rightarrow \frac{d\lambda}{\lambda} = -\left(\frac{\sigma}{\varepsilon}\right) dt$$

$$\Rightarrow \lambda = \lambda_0 e^{-(\sigma/\varepsilon)t}$$

So, $j = \frac{I}{2\pi r \ell} = \frac{\lambda\sigma}{2\pi\varepsilon r} = \left(\frac{\lambda_0 \sigma}{2\pi\varepsilon r}\right) e^{-(\sigma/\varepsilon)t}$



ONE OR MORE THAN ONE CHOICE CORRECT

6. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n >> 1. Which of the following statement(s) is (are) true?

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive orbitals varies as 1/n

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

Ans. (ABD)

$$r_n = \frac{n^2}{Z} a_0$$

$$\Delta r_n = \frac{2na_0}{Z}$$

$$\frac{\Delta r_n}{r_n} = \frac{2}{n}$$

$$E_n = \frac{-13.6Z^2}{n^2}$$

$$\Delta E_n = \frac{13.6 \times 2 \times Z^2}{n^3}$$

so, $\frac{\Delta E_n}{E_n} = -\frac{2}{n}$

$$L_n = \frac{nh}{2\pi}$$

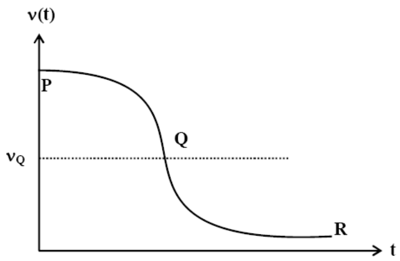
$$\frac{\Delta L_n}{L_n} = \frac{1}{n}.$$

7. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let ν(t) represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

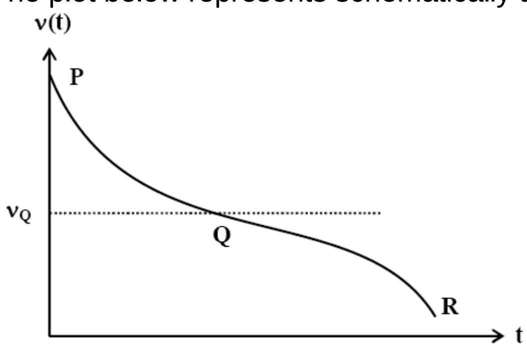
(A) $\nu_P + \nu_R = 2\nu_Q$

(B) The rate of change in beat frequency is maximum when the car passes through Q

(C) The plot below represents schematically the variation of beat frequency with time



(D) The plot below represents schematically the variation of beat frequency with time



Ans. (ABC)

$$\nu_P = (121-118)\left(\frac{v + v_D\cos\alpha}{v}\right) = 3\left(\frac{v + v_D\cos\alpha}{v}\right)\text{Hz}$$

ν_Q = 121 – 118 = 3Hz

$$\nu_R = (121-118)\left(\frac{v - v_D\cos\alpha}{v}\right) = 3\left(\frac{v - v_D\cos\alpha}{v}\right)$$

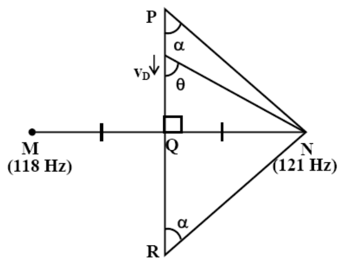
$\therefore$ $\nu_P + \nu_R = 2\nu_Q$

Now, ν (between P and Q) = $3\left(\frac{v + v_D\cos\theta}{v}\right)$

$$\frac{d\nu}{d\theta} = -\frac{3v_D}{v}\sin\theta$$

Now, sin θ = 1 at Q (maximum)

$\therefore$ rate of change in beat frequency is maximum at Q



8. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?
(A) The temperature distribution over the filament is uniform
(B) The resistance over small sections of the filament decreases with time
(C) The filament emits more light at higher band of frequencies before it breaks up
(D) The filament consumes less electrical power towards the end of the life of the bulb

Ans. (C, D)
When filament breaks up, the temperature of filament will be higher so according to wein's law $\left(\lambda_m \propto \frac{1}{T}, v_m \propto T\right)$, the filament emits more light at higher band of frequencies.

As voltage is constant, so consumed electrical power is $P = \frac{V^2}{R}$

As R increases with increase in temperature so the filament consumes less electrical power towards the end of the life of the bulb.

9. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?
(A) The refractive index of the lens is 2.5
(B) The radius of curvature of the convex surface is 45 cm
(C) The faint image is erect and real
(D) The focal length of the lens is 20 cm

Ans. (A, D)
For refraction through lens,
$\frac{1}{v} - \frac{1}{-30} = \frac{1}{f}$ and $-2 = \frac{v}{u}$
$\therefore$ v = –2u = 60 cm
$\therefore$ f = +20 cm
For reflection
$\frac{1}{10} + \frac{1}{-30} = \frac{2}{R} \Rightarrow R = 30\,\text{cm}$
$(n-1)\left(\frac{1}{R}\right) = \frac{1}{f} = \frac{1}{20}$
$\therefore\ n = \frac{5}{2}$
The faint image is erect and virtual.

10. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for ℓ is(are) dimensionally correct?

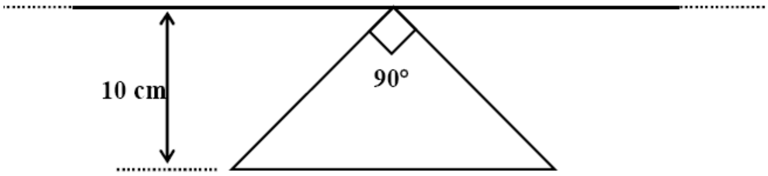
(A) $\ell = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (B) $\ell = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$ (C) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

Ans. (BD)

$k_B T \approx FL$, $\varepsilon \approx \frac{Q^2}{FL^2}$ and $n \approx L^{-3}$; where F is force, Q is charge and L is length

So (B) and (D) are correct

11. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true?



(A) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(B) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(C) The induced current in the wire is in opposite direction to the current along the hypotenuse

(D) There is a repulsive force between the wire and the loop

Ans. (AD)

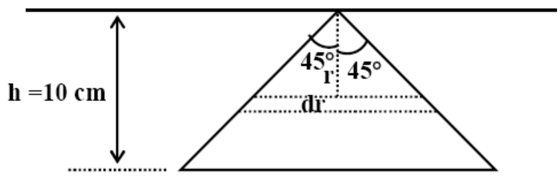
$$\phi_{\ell w} = \int_0^h \frac{\mu_0 I}{2\pi r} 2r dr = \frac{\mu_0 I h}{\pi}$$

So, Mutual inductance $M_{\ell w} = \frac{\mu_0 h}{\pi}$

$$\therefore \quad \varepsilon_w = \frac{\mu_0 h}{\pi}\frac{di}{dt} = \frac{\mu_0}{\pi}$$

Due to rotation there is no change in flux through the wire, so there is no extra induced emf in the wire.

From Lenz's Law, current in the wire is rightward so repulsive force acts between the wire and loop.



12. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$, where $\alpha = 10/3\,ms^{-3}$, $\beta = 5\,ms^{-3}$, and m = 0.1 kg. At = t = 1s, which of the following statement(s) is (are) true about the particle?

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})ms^{-1}$

(B) The angular momentum $\vec{L}$ is with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}\,Nms$

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})N$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\hat{k}\,Nm$

Ans. (ABD)

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = 3\alpha t^2\hat{i} + 2\beta t\hat{j},\ \vec{a} = \frac{d\vec{v}}{dt} = 6\alpha t\hat{i} + 2\beta\hat{j}$$

At t = 1 s, $\vec{v} = \left(10\hat{i} + 10\hat{j}\right)ms^{-1}$

$$\vec{a} = 20\hat{i} + 10\hat{j}\,ms^{-2}$$

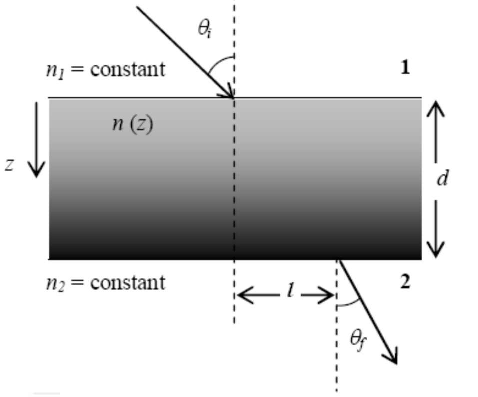
$$\vec{r} = \frac{10}{3}\hat{i} + 5\hat{j}\,m$$

$$\vec{L}_0 = \vec{r} \times m\vec{v} = \left(-\frac{5}{3}\hat{k}\right)Nms$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = \left(2\hat{i} + \hat{j}\right)N$$

$$\vec{\tau}_0 = \vec{r} \times \vec{F} = \vec{r} \times m\vec{a} = \left(-\frac{20}{3}\hat{k}\right)Nm$$

13. A transparent slab of thickness d has a refractive index n (z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 (> n_1), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement (s) is (are) true?

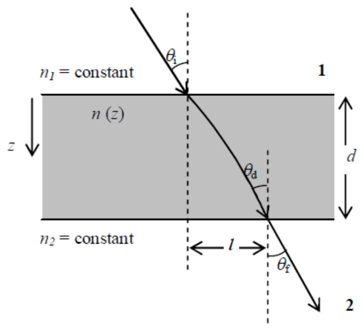
(A) $n_1 \sin\theta_I = n_2 \sin\theta_f$

(B) $n_1 \sin\theta_I = (n_2 - n_1) \sin\theta_f$

(C) l is independent of n_2

(D) l is dependent on n (z)

Ans. (ACD)



From Snell's Law

$$n_1 \sin\theta_i = n(d)\sin\theta_d = n_2 \sin\theta_f$$

The deviation of ray in the slab will depend on n (z)

Hence, l will depend on n(z) but not on n_2.

Integer type

14. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2 (P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

Ans. (9)

Power radiated $P = e\sigma AT^4$

At 487°C; $P_1 = e\sigma A(760)^4$ ...(i)

Given $\log_2 \frac{P_1}{P_0} = 1 \Rightarrow P_0 = \frac{P_1}{2}$

At $2767^oC; P_2 = e\sigma A(3040)^4$

$\therefore$ Reading $= \log_2\left(\frac{P_2}{P_0}\right) = \log_2\left[\frac{e\sigma A(3040)^4 \times 2}{e\sigma A(760)^4}\right] = \log_2(4^4 \times 2) = 9$

15. The isotope ${}^{12}_{5}B$ having a mass 12.0.14 u undergoes β-decay to ${}^{12}_{6}C$. ${}^{12}_{6}C$ has an excited state of the nucleus (${}^{12}_{6}C^*$) at 4.041 MeV above its its ground state. If ${}^{12}_{5}B$ decays to ${}^{12}_{5}C^*$, the maximum kinetic energy of the β- particle in units of MeV is ($1u = 931.5$ MeV / c^2, where c is the speed of light in vacuum).

Ans. (9)

Q value $= \left[12.014u - \left(12u + 4.041\frac{MeV}{c^2}\right)\right]c^2$

$Q = (0.014u \times 931.5)MeV - (4.041)MeV = 9MeV$

Hence, β particle will have a maximum KE of 9 MeV

16. A hydrogen atom in its ground state is irradiated by light of wavelength $970\,\overset{o}{A}$. Taking $hc/e = 1237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is

Ans. (6)

Note : Unit of $\frac{hc}{e}$ in original paper is incorrect

Energy available $E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6} eVm}{970 \times 10^{-10} m} = 12.75 eV$ (4^{th} energy level of hydrogen atom)

Hence, the number of lines present in the emission spectrum $= {}^4C_2 = 6$

17. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm. respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

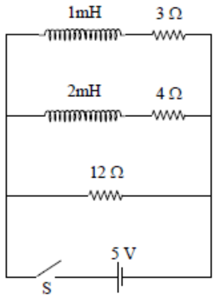
Ans. (3)

Terminal velocity $v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g$ where ρ is the density of the solid sphere and σ is the density of the liquid

$$\therefore \frac{v_P}{v_Q} = \frac{(8-0.8) \times \left(\frac{1}{2}\right)^2 \times 2}{(8-1.6) \times \left(\frac{1}{4}\right)^2 \times 3} = 3$$

18. Two inductors L_1 (inductance 1 mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current ((I_{max}/I_{min}) drawn from the battery is

Ans. (8)



At t = 0, current will flow only in 12Ω resistance

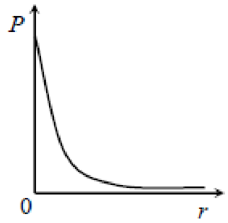
$\therefore\ I_{min} = \frac{5}{12}$

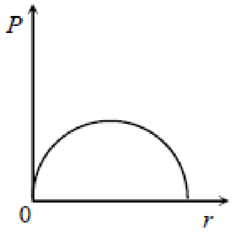
At t $\rightarrow \infty$ both L_1 and L_2 behave as conducting wires

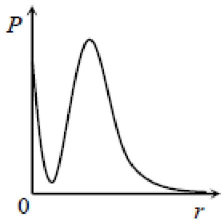
$\therefore\ R_{eff} = \frac{3}{2}$

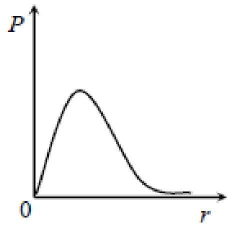
PART II: CHEMISTRY

19. P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is

(A) 

(B) 

(C) 

(D) 

Ans. (D)

The probability distribution curve for 1 s electron of hydrogen atom.

20. One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surrounding (ΔS_{surr}) in JK^{-1} is (1 L atm = 101.3 J)

(A) 5.763 (B) 1.013 (C) –1.013 (D) –5.763

Ans. (C)

Isothermal process, $\Delta U = 0$

$dq = -dW = P_{ext}(V_2 - V_1) = 3$ L – atm $= 3 \times 101.3$ Joule

$$\Delta S_{surrounding} = \frac{3 \times 101.3}{300} \text{Joule} K^{-1} = -1.013 \text{ Joule } K^{-1}$$

$\therefore\ \Delta S_{surr} = -1.013 \text{ Joule } K^{-1}$

21. The increasing order of atomic radii of the following Group 13 elements is

(A) Al < Ga < In < Tl (B) Ga < Al < In < Tl

(C) Al < In < Ga < Tl (D) Al < Ga < Tl < In

Sol. (C)

Isothermal process, $\Delta U = 0$

$dq = -dW = P_{ext}(V_2 - V_1) = 3L - atm = 3 \times 101.3 Joule$

$$\Delta S_{surrounding} = -\frac{3 \times 101.3}{300} \text{ Joule } K^{-1} = -1.013 \text{ Joule } K^{-1}$$

$\therefore\ \Delta S_{surr} = -1.013 \text{ Joule } K^{-1}$

21. The increasing order of atomic radii of the following Group 13 elements is

(A) Al < Ga < In < Tl (B) Ga < Al < In < Tl (C) Al < In < Ga < Tl (D) Al < Ga < T1 < In

Sol. (B)

Increasing order of atomic radius of group 13 elements Ga < Al < In < Tl.

Due to poor shielding of d-electrons in Ga, its radius decreases below Al.

22. Among $[Ni(CO)_4], [NiCl_4]^{2-}, [Co(NH_3)_4Cl_2]Cl, Na_3[CoF_6]$, Na_2O_2 and CsO_2, the total number of paramagnetic compounds is

(A) 2 (B) 3 (C) 4 (D) 5

Sol. (B)

Number of paramagnetic compounds are 3.

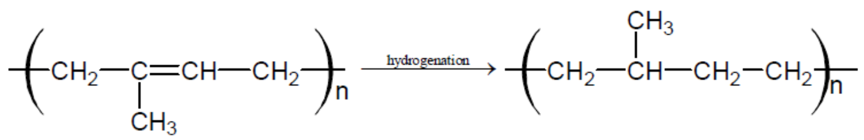
Following compounds are paramagnetic. $[NiCl_4]^{2-}, Na_3[CoF_6], CsO_2$

23. On complete hydrogenation, natural rubber produces

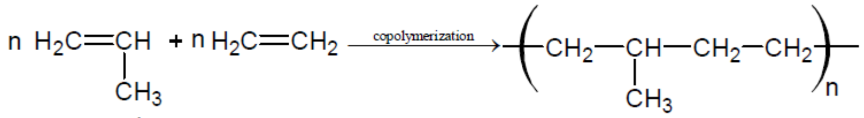
(A) ethylene-propylene copolymer (B) vulcanised rubber

(C) polypropylene (D) polybutylene

Sol. (A)



NR



propylene

propylene-ethylene copolymer

ONE OR MORE THAN ONE CHOICE CORRECT

24. According to the Arrhenius equation,
(A) a high activation energy usually implies a fast reaction.
(B) rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.
(C) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant.
(D) the pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

Sol. (B,C,D)

25. A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, Z > 20. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is(are)
(A) β^- -decay (β emission) (B) orbital or K-electron capture
(C) neutron emission (D) β^+ -decay (positron emission)

Sol. (B, D)

$\frac{n}{p} < 1$

Positron, emission ${}_{a}X^{b} \rightarrow {}^{0}_{+1}e + {}_{a-1}Y^{b}$

$\Rightarrow$ K – electron capture

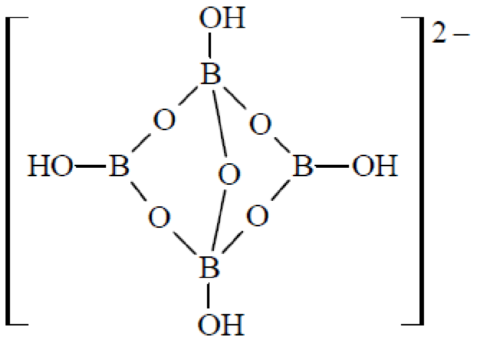
${}_{a}X^{b} + {}_{-1}e^{0} \rightarrow {}_{a-1}Y^{b}$

Both process cause an increase in n/p ratio towards 1 thus stabilising the nucleus

26. The crystalline form of borax has
(A) tetranuclear $[B_4O_5(OH)_4]^{2-}$ unit
(B) all boron atoms in the dame plane
(C) equal number of sp^2 and sp^3 hybridized boron atoms
(D) One terminal hydroxide per boron atom

Sol. (A, C, D)

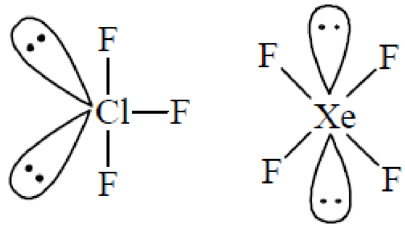
The structure of anion of borax is



27. The compound(s) with TWO lone pairs of electrons on the central atom is (are)
(A) BrF_5 (B) ClF_3 (C) XeF_4 (D) SF_4

Sol. (B, C)

ClF_3 and XeF_4 contain two lone pair of electrons on the central atom.



28. The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is (are)

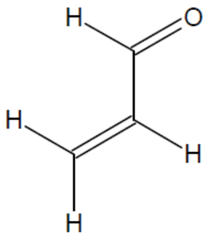
(A) $CuCl_2$ (B) $BaCl_2$ (C) $Pb(OOCCH_3)_2$ (D) $Na_2[Fe(CN)_5NO]$

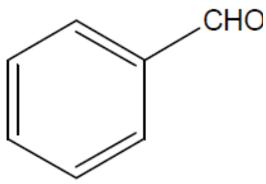
Sol. (A)

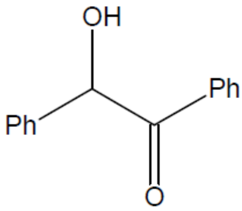
The reagent that can selectively precipitate S^{2-} and SO_4^{2-} in aqueous solution is $CuCl_2$.

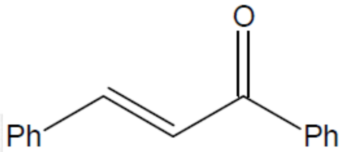
$$S^{2-} + CuCl_2 \rightarrow CuS\downarrow + 2Cl^-$$

29. Positive Tollen's test is observed for

(A) 

(B) 

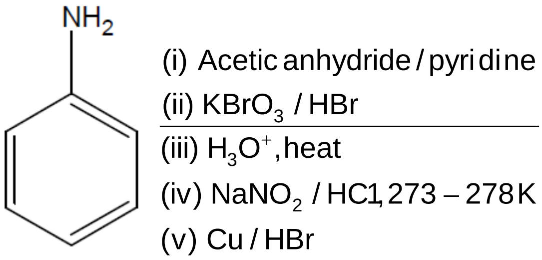
(C) 

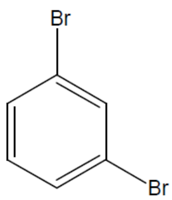
(D) 

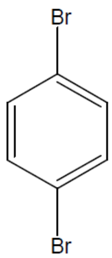
Sol. (A, B)

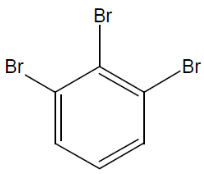
Beside aldehyde $\alpha-$hydroxy ketones can also show Tollen's test due to rearrangement in aldehyde via ene diol intermediate. (however this needs a terminal $\alpha-$carbon)

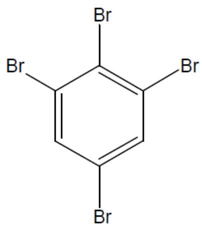
30. The product(s) of the following reaction sequence is (are)



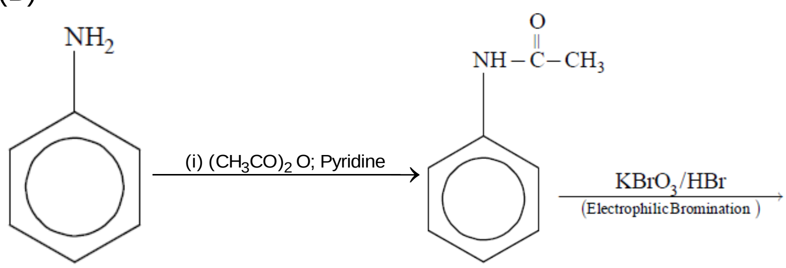
(A) 

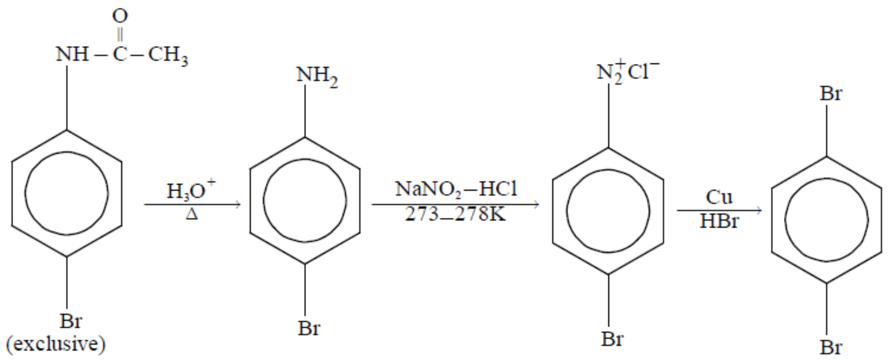
(B) 

(C) 

(D) 

Sol. (B)





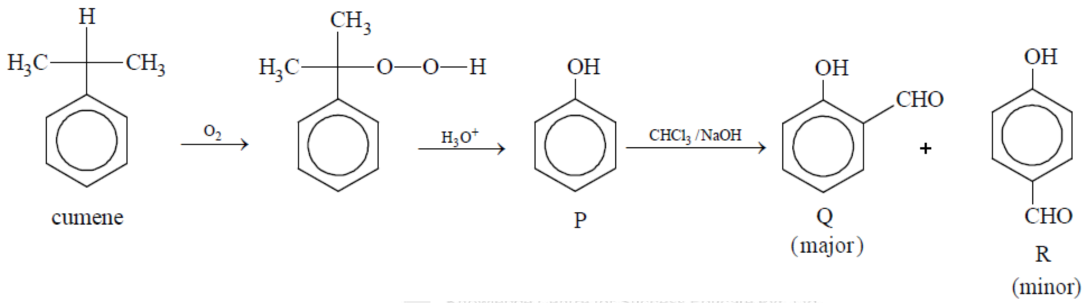
31. The correct statement(s) about the following reaction sequence is (are)

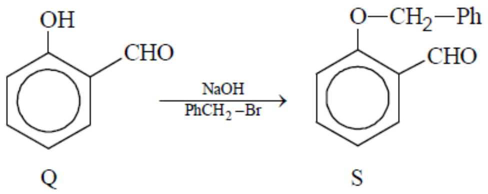
$$\text{Cumene } (C_9H_{12}) \xrightarrow[\text{ii)}H_3O^+]{\text{i)}O_2} P \xrightarrow{CHCl_3/NaOH} Q(\text{major}) + R(\text{minor})$$

$$\mathbf{Q} \xrightarrow[PhCH_2Br]{NaOH} \mathbf{S}$$

(A) R is steam volatile
(B) Q gives dark violet coloration with 1% aqueous $FeCl_3$ solution
(C) S gives yellow precipitate with 2, 4-dinitrophenylhydrazine
(D) S gives dark violet coloration with 1% aqueous $FeCl_3$ solution

Sol. (B, C)





Q (not R) is steam volatile due to intermolecular hydrogen bonding.
Q gives violet colouration due to phenolic functional group.
S gives yellow precipitate due to aldehydic group with 2, 4 –DNP.
S does not have free phenolic group to respond to $FeCl_3$ test.

Integer type

32. The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{sohute}}{MW_{solvent}}\right)$, is

Sol. (9)

$$m = \frac{X_A \times 1000}{X_B \times M_A}$$

$$m = \frac{1000}{9M_A} \qquad ...(1)$$

$$M = \frac{n_B \times 1000 \times d}{n_A \times M_A + n_B \times M_B} = \frac{X_B \times 1000 \times d}{X_A \times M_A + X_B \times M_B}$$

$$= \frac{200}{0.9M_A + 0.1M_B}$$

$$= \frac{2000}{9M_A + M_B} \quad \text{...(ii)}$$

As m = M

$$\frac{1000}{9M_A} = \frac{2000}{9M_A + M_B}$$

$$9M_A + M_B = 18M_A$$

$$\therefore \; 9M_A = M_B$$

$$\therefore \frac{M_B}{M_A} = 9$$

33. The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases *x* times. The value of *x* is

Sol. (4)

Mean free path $(\ell)\alpha\frac{T}{P}$

Mean speed $(C_{av})\alpha\sqrt{T}$

Diffusion coefficient (D) is proportional to both mean free path (ℓ) and mean speed

$$\therefore \; D\alpha\frac{T^{3/2}}{P}$$

At temperature T_1 and P_1

$$D_1 = \frac{KT_1^{3/2}}{P_1} \quad \text{...(1)}$$

If T_1 is increased 4 times and P_1 is increased 2 times

$$D_1 = \frac{K(4T_1)^{3/2}}{2P_1} \quad \text{...(2)}$$

$$\frac{D_2}{D_1} = \frac{(4)^{3/2}}{2} = \frac{2^3}{2} = 4$$

34. In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is

Sol. (6)

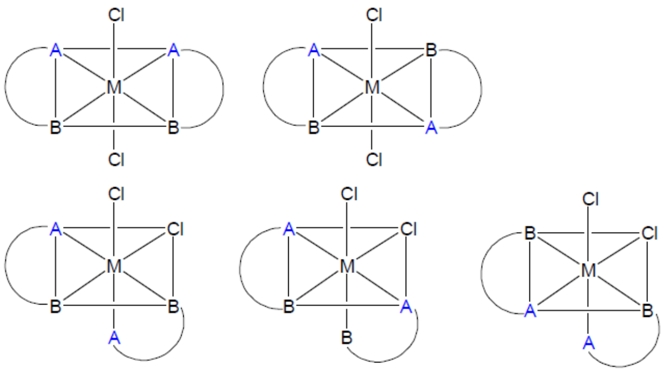
$$8MnO_4^- + 3S_2O_3^{2-} + H_2O \xrightarrow{\text{faintlyalkakine}} 8MnO_2 + 6SO_4^{2-} + 2OH^-$$

$\therefore$ 8 moles MnO_4^- produce 6 moles SO_4^{2-}

35. The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-(L = H_2NCH_2CH_2O^-)$ is

Sol. (5)

Number of geometrical isomers = 5

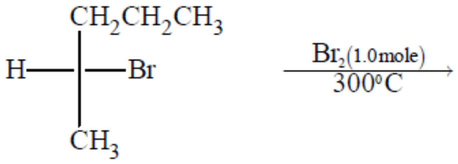


$AB = H_2NCH_2CH_2O^-$

A site is N

B site is O

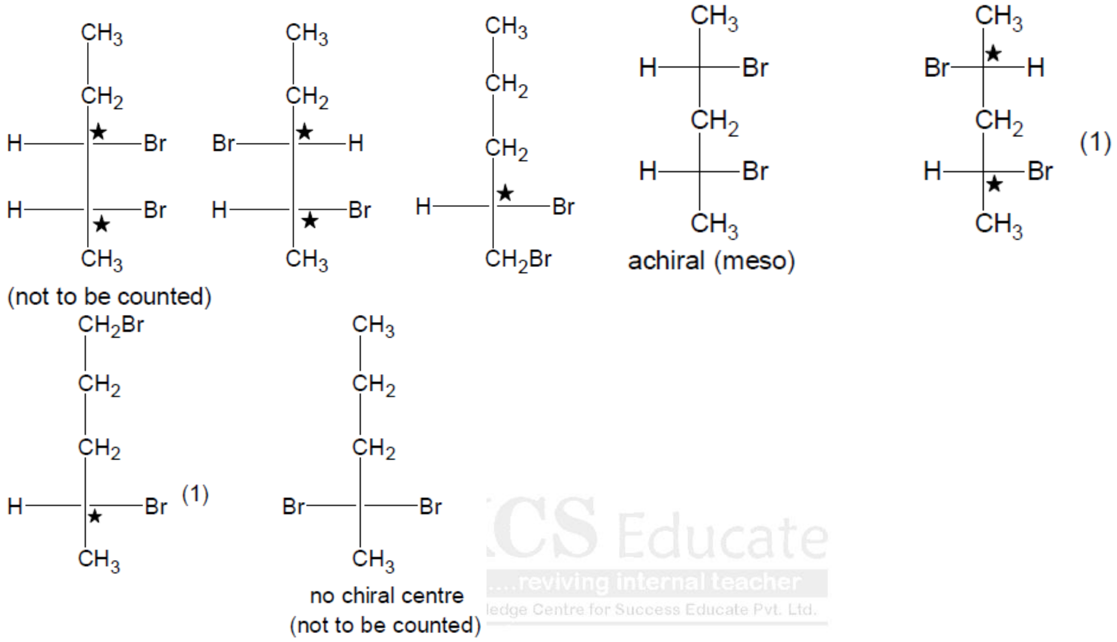
36. In the following monobromination reaction, the number of possible chiral products is



(1.0 mole)

(enantiomerically pure)

Sol. (5)



PART III: MATHEMATICS

37. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$ then $\alpha_1 + \beta_2$ equals

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Sol. (C)

$(\alpha_1, \beta_1) = \sec\theta \pm \tan\theta$

Since $\alpha_1 > \beta_1$

$\alpha_1 = \sec\theta - \tan\theta$

$\beta_1 = \sec\theta + \tan\theta$

$(\alpha_2, \beta_2) = -\tan\theta \pm \sec\theta$

Since $\alpha_2 > \beta_2$

$\alpha_2 = -\tan\theta + \sec\theta$

$\beta_2 = -\tan\theta - \sec\theta$

Hence, $\alpha_1 + \beta_2 = -2\tan\theta$

38. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320 (C) 260 (D) 95

Sol. (A)

If a boy is selected then number of ways $= {}^4C_1 . {}^6C_3$

If a boy is not selected then number of ways $= {}^6C_4$

Captain can be selected in ${}^4C_1$ ways

Required number of ways $= {}^4C_1 . {}^6C_3 . {}^4C_1 + {}^6C_4 . {}^4C_1 = 380$

39. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation

$\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

Sol. (C)

$\left(\sqrt{3}\sin x + \cos x\right) = 2\cos 2x$

$\cos\left(x - \frac{\pi}{3}\right) = \cos 2x$

$x - \frac{\pi}{3} = 2n\pi \pm 2x$

$\Rightarrow x = -\frac{\pi}{3}, -\frac{5\pi}{9}, \frac{\pi}{9}, \frac{7\pi}{9}$

$\Rightarrow \sum x_i = 0$

40. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P(computer turns out to be defective given that it is produced in plant T_1)

= 10 P(computer turns out to be defective given that it is produced in plant T_2),

where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Sol. (C)

E_1: Computer is produced by plant T_1

E_2 : Computer is produced by plant T_2

A : Computer is defective

Now, $P(A/E_1) = 10P(A/E_2)$

$\Rightarrow \dfrac{P(A \cap E_1)}{P(A \cap E_2)} = \dfrac{5}{2}$

Let $P(E_2 \cap A) = \dfrac{80 - x}{100}$

$\Rightarrow P(E_2 \cap A) = \dfrac{80 - x}{100}$

$P(E_1 \cap A) = \dfrac{x - 73}{100}$

$\Rightarrow x = 78$

$\Rightarrow P(E_2 / \bar{A}) = \dfrac{P(E_2 \cap \bar{A})}{P(\bar{A})} = \dfrac{78}{93}$

41. The least value of $\alpha \in \mathbb{R}$ for which $4\alpha x^2 + \dfrac{1}{x} \ge 1$, for all x > 0, is

(A) $\dfrac{1}{64}$ (B) $\dfrac{1}{32}$ (C) $\dfrac{1}{27}$ (D) $\dfrac{1}{25}$

Sol. (C)

$4\alpha x^2 + \dfrac{1}{x} \ge 1, x > 0$

Let $f(x) = 4\alpha x^2 + \dfrac{1}{x} \Rightarrow f'(x) = 8\alpha x - \dfrac{1}{x^2}$

$f'(x) = 0$ at $x = \dfrac{1}{2\alpha^{1/3}}$

$3\alpha^{1/3} = 1 \Rightarrow \alpha = \dfrac{1}{27}$

Alternate

$\dfrac{4\alpha x^2 + \dfrac{1}{2x} + \dfrac{1}{2x}}{3} \ge (\alpha)^{1/3}$

$\Rightarrow 4\alpha x^2 + \dfrac{1}{x} \ge 3(\alpha)^{1/3}$

$\Rightarrow 3\alpha^{1/3} = 1$

$\alpha = \dfrac{1}{27}$

ONE OR MORE THAN ONE CHOICE CORRECT

42. Consider a pyramid OPQRS located in the first octant $(x \ge 0, y \ge 0, z \ge 0)$ with O as origin, and OR along the x-axis and the y-axis, respectively. The bases OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then

(A) the acute angle between OQ and OS is $\dfrac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is x – y = 0

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\dfrac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\dfrac{15}{2}}$

Sol. (B, C,D)

Points O, P, Q, R, S are (0, 0, 0), (3, 0, 0), (3, 3, 0) (0, 3, 0), $\left(\frac{3}{2},\frac{3}{2},3\right)$ respectively.

$\Rightarrow$ Angle between OQ and OS is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Equation of plane containing the points O, Q and S is x – y = 0

$\Rightarrow$ Perpendicular distance from P(3, 0, 0) to the plane

$x-y=0$ is $\left|\frac{3-0}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$

Perpendicular distance from O(0, 0, 0) to the line RS

$\frac{x}{1}=\frac{y-3}{-1}=\frac{z}{2}$ is $\sqrt{\frac{15}{2}}$

43. Let $f:(0,\infty)\to\mathbb{R}$ be a differentiable function such that $f'(x)=2-\frac{f(x)}{x}$ for all $x\in(0,\infty)$ and $f(1)\neq 1$. Then

(A) $\lim\limits_{x\to 0+} f'\left(\frac{1}{x}\right)=1$ (B) $\lim\limits_{x\to 0+} xf\left(\frac{1}{x}\right)=2$ (C) $\lim\limits_{x\to 0+} x^2f'(x)=0$ (D) $|f(x)|\leq 2$ for all $x\in(0,2)$

Sol. (A)

$f'(x)+\frac{1}{x}f(x)=2 \quad \forall x\in(0,\infty)$

$\Rightarrow f(x)=x+\frac{c}{x}, c\neq 0$ as $f(1)\neq 1$

(A) $\lim\limits_{x\to 0^+} f'\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^+}(2-1-cx^2)=1$

(B) $\lim\limits_{x\to 0^+}\left(xf\left(\frac{1}{x}\right)\right)=\lim\limits_{x\to 0^+}(1+cx^2)=1$

(C) $\lim\limits_{x\to 0^+} x^2 f'(x)=\lim\limits_{x\to 0^+}(x^2-c)=-c\neq 0$

(D) for $c\neq 0\, f(x)$ is unbounded function for $x\in(0,2)$

44. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in\mathbb{R}$. Suppose $Q=[q_{ij}]$ is a matrix such that PQ = kI, where $k\in\mathbb{R}$, $k\neq 0$ and I is the identity of order 3. If $q_{23}=-\frac{k}{8}$ and det $(Q)=\frac{k^2}{2}$, then

(A) $\alpha=0, k=8$

(B) $4\alpha-k+8=0$

(C) det (P adj(Q)) = 2^9

(D) det(Q adj(P)) = 2^{13}

Sol. (B, C)

$P\left(\frac{Q}{k}\right)=I$

$P^{-1}=\left(\frac{Q}{k}\right)$

$(P^{-1})_{23}=\frac{q_{23}}{k}=-\frac{1}{8}$

$-\frac{(3\alpha+4)}{20+12\alpha}=-\frac{1}{8}\ \alpha=-1$

$\Rightarrow$ det(P) = 20 + 12α = 8

$$(\det P)\left(\det\left(\frac{Q}{k}\right)\right)=1$$

$$\frac{8\det(Q)}{k^3}=1 \Rightarrow |Q| = \frac{k^3}{8}$$

$$\Rightarrow \frac{k^3}{8}=\frac{k^2}{2} \Rightarrow k = 4 \Rightarrow \det(Q) = 8$$

det (P.adjQ) = detP. det adjQ

$= \det P(\det Q)^2 = 8 \times 8^2 = 2^9$

det Q . adj P = $\det Q(\det P)^2 = 8 \times 8^2 = 2^9$.

Alternate

$$|P|\cdot|Q|=k^3 \Rightarrow |P|=2k$$

$$\Rightarrow 6\alpha+10=k \qquad ...(1)$$

Also PQ = kI

|P|Q = k adj(P)

2kQ = k adj(P)

Comparing q_{23} we get

$$-\frac{k}{4}=-3\alpha-4 \qquad ...(2)$$

Solving (1) and (2) we get $\alpha=-1$ and k = 4

45. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z respectively, and $2s=x+y+z$. If $\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}=\frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right)=\frac{3}{5}$

Sol. (A, C, D)

$$\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}=\frac{s}{9}$$

$$\Rightarrow \Delta=\frac{2\sqrt{6}}{27}s^2 \text{ and inradius } r=\frac{2\sqrt{6}}{27}s$$

$\Rightarrow$ s = 9, x = 5, y = 6, z = 7

$$R=\frac{35}{24}\sqrt{6}$$

$$r=4R\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}$$

$$\Rightarrow \sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}=\frac{4}{35}$$

$$\sin^2\left(\frac{X+Y}{2}\right)=\frac{1}{2}(1-\cos(X+Y))=\frac{1}{2}(1+\cos Z)=\frac{1}{2}\left(1+\frac{1}{5}\right)=\frac{3}{5}$$

46. A solution curve of the differential equation $\left(x^2+xy+4x+2y+4\right)\frac{dy}{dx}-y^2=0, x>0,$ passes through the point (1, 3). Then the solution curve

(A) intersects y = x + 2 exactly at one point
(B) intersects y = x + 2 exactly at two points
(C) intersects $y = (x + 2)^2$
(D) does NOT intersect $y=(x+3)^2$

Sol. (A, D)

$$\left[(x+2)^2+y(x+2)\right]\frac{dy}{dx}-y^2=0$$

$$\frac{dy}{y}=\frac{yd(x+2)-(x+2)dy}{(x+2)^2}$$

$$\Rightarrow \ln y=-\frac{y}{x+2}+c,\ c = 1 + \ln 3 \quad \text{since } y(1) = 3$$

$$\Rightarrow \ln\left(\frac{y}{3}\right)+\frac{y}{x+2}=1$$

For y = x + 2, $\ln\left(\frac{x+2}{3}\right)=0$

$\Rightarrow$ x = 1 only for $y=(x+2)^2$

$$\ln\left(\frac{(x+2)^2}{3}\right)+(x+2)=1$$

$$\frac{(x+2)^2}{3}>\frac{4}{3}>1 \quad \forall x>0$$

$$\Rightarrow \ln\left(\frac{(x+2)^2}{3}\right)+(x+2)>2 \qquad \forall x>0$$

Hence no solution

47. Let $f:\mathbb{R}\to\mathbb{R}, g:\mathbb{R}\to\mathbb{R}$ and $h:\mathbb{R}\to\mathbb{R}$ be differentiable functions such that $f(x)=x^3+3x+2$. $g(f(x))=x$ and $h(g(g(x)))=x$ for all $x\in\mathbb{R}$. Then

(A) $g'(2)=\frac{1}{15}$ (B) $h'(1)=666$ (C) $h(0)=16$ (D) $h(g(3))=36$

Sol. (B, C)

$g'(f(x)).f'(x)=1$

$\Rightarrow g'(2)=\frac{1}{3}$

h(x) = f(f(x))

$\Rightarrow$ h(0) = 16

h(x) = f(f(x))

$\Rightarrow h'(x) = f'(f(x))f'(x)$

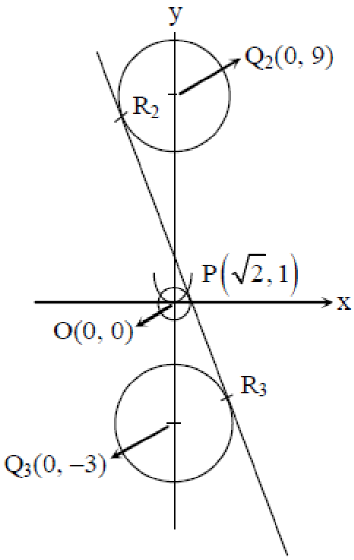
$\Rightarrow h'(1) = f'(f(1)).f'(1) = 111 \times 6 = 666$

48. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3=12$ (B) $R_2R_3=4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Sol. (A, B, C)



Equation of tangent at $P\left(\sqrt{2},1\right)$ is $\sqrt{2}x+y-3=0$

If centre of C_2 at (0, α) and radius equal to $2\sqrt{3}$

$2\sqrt{3}=\left|\frac{\alpha-3}{\sqrt{3}}\right| \Rightarrow \alpha=-3,9$

(A) $Q_2Q_3=12$

(B) $R_2R_3=$ length of transverse common tangent

$=\sqrt{(Q_2Q_3)^2-(r_1+r_2)^2}=\sqrt{(12)^2-(2\sqrt{3}+2\sqrt{3})^2}=4\sqrt{6}$

(C) Area of ΔOR_2R_3

$=\frac{1}{2}\times R_2R_3\times$ perpendicular distance of O from line

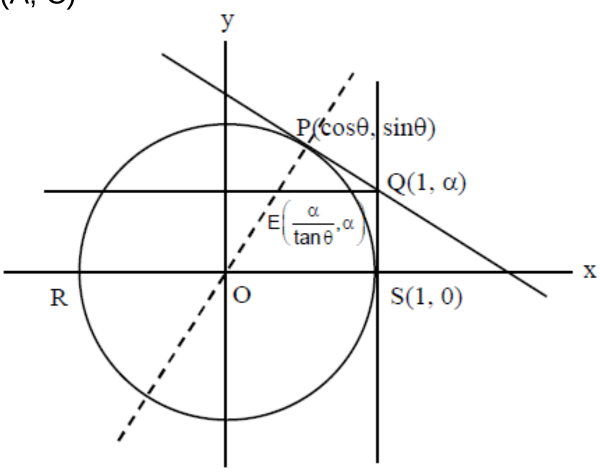
$=\frac{1}{2}\times 4\sqrt{6}\times\sqrt{3}=6\sqrt{2}$

(D) Area of $\Delta PQ_2Q_3=\frac{1}{2}\times 12\times\sqrt{2}=6\sqrt{2}$

49. Let RS be the diameter of the circle $x^2+y^2=1$, where S is the point (1, 0). Let P be a variable point (other the R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4},\frac{1}{2}\right)$ (C) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4},-\frac{1}{2}\right)$

Sol. (A, C)



$E \equiv \left[\dfrac{\alpha}{\tan\theta}, \alpha\right]$

$\cos\theta + \alpha\sin\theta = 1$

$\alpha = \tan\dfrac{\theta}{2}$

$\therefore$ locus is $y^2 = 1 - 2x$

Integer type

50. The total number of distinct $x \in \mathbb{R}$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

Sol. (2)

$$x^3\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

$$x^3\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} + x^6\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 3 & 6 & 24 \end{vmatrix} = 10$$

$x^3(-4+6) + x^6(48-36) = 10$

$2x^3 + 12x^6 = 10$

$6x^6 + x^3 - 5 = 0$

$6x^6 + 6x^3 - 5x^3 - 5 = 0$

$(6x^3 - 5)(x^3 + 1) = 0$

$x^3 = \dfrac{5}{6}, x^3 = -1 \Rightarrow x = \left(\dfrac{5}{6}\right)^{1/6}, x = -1$

51. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + ... + (1+x)^{49} + (1+mx)^{50}$ is $(3n+1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is

Sol. (5)

Here $(3n+1)\,{}^{51}C_3 = \left(\text{coefficient of } x^2 \text{ in } (1+x)^2\dfrac{\left[(1+x)^{48} - 1\right]}{1+x-1}\right) + {}^{50}C_2 m^2$

$(3n+1)\,{}^{51}C_3 = \left(\text{coefficient of } x^3 \text{ in } (1+x)^{50} - (1+x)^2\right) + {}^{50}C_2 m^2$

$\Rightarrow (3n+1)\,{}^{51}C_3 = {}^{50}C_3 + {}^{50}C_2 m^2 \Rightarrow n = \dfrac{m^2 - 1}{51}$

Least positive integer m for which n is an integer is m = 16 for which n = 5

52. The total number of distinct $x \in [0,1]$ for which $\int_0^x \dfrac{t^2}{1+t^4}dt = 2x - 1$ is

Sol. (1)

$\int_0^x \dfrac{t^2}{1+t^4}dt = 2x - 1$

Let $f(x) = \int_0^x \dfrac{t^2}{1+t^4}$ dt = 2x + 1

$\Rightarrow f'(x) = \dfrac{x^2}{1+x^4} - 2 < 0 \ \forall x \in [0,1]$

Now, $f(0)=1$ and $f(1)=\int_0^1 \frac{t^2}{1+t^4}dt-1$

As $0 \le \frac{t^2}{1+t^4} < \frac{1}{2} \qquad \forall t \in [0,1]$

$\Rightarrow \int_0^1 \frac{t^2}{1+t^4}dt < \frac{1}{2} \Rightarrow$ f(1) < 0

$\Rightarrow$ f(x) = 0 has exactly one root in [0, 1].

53. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x\to 0} \frac{x^2 \sin\beta x}{\alpha x - \sin x} = 1$. Then $6(\alpha+\beta)$ equals

Sol. (7)

$$\lim_{x\to 0} \frac{x^2 \sin\beta x}{\alpha x - \sin x} \times \frac{\beta x}{\beta x} = 1$$

$$\lim_{x\to 0} \frac{x^3\beta}{\alpha x - \left[x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5}\right]} = 1$$

$$\lim_{x\to 0} \frac{x^3\beta}{(\alpha-1)x + \frac{x^3}{\lfloor 3} - \frac{x^5}{\lfloor 5} + \ldots\ldots} = 1$$

$$\alpha - 1 = 0, \beta \times 3! = 1, \beta = \frac{1}{6}$$

$$\alpha = 1$$

$$6(\alpha+\beta) = 6\left(1+\frac{1}{6}\right) = 7$$

54. Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i=\sqrt{-1}$, and r, $s \in \{1,2,3\}$, Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is

Sol. (1)

Z = ω

$$P = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ (\omega)^{2s} & \omega^r \end{bmatrix}$$

$$P^2 = \begin{bmatrix} (-\omega)^r & (\omega)^{2s} \\ (\omega)^{2s} & \omega^r \end{bmatrix}\begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} = \begin{bmatrix} (-\omega)^{2r} + \omega^{4s} & (-\omega)^r(\omega)^{2s} + \omega^{2s}\omega^r \\ (-\omega)^r\omega^{2s} + \omega^r\omega^{2s} & \omega^{4s} + \omega^{2r} \end{bmatrix}$$

$$P^2 = -I$$

$\omega^{2r} + \omega^{4s} = -1$ and $\omega^{2s}(-\omega)^r + \omega^{2s}\omega^r = 0$

$$\omega^{2r} + \omega^s = -1$$

$$1 + \omega^s + \omega^{2r} = 0$$

r = s = 1